California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 3: Kirchhoff's Voltage and Current Laws

Objective

The objective of this experiment is to make an experimental demonstration of Kirchhoff's Voltage Law and Kirchhoff's Current Law. This will be done for a circuit with a DC source and then for a circuit with an AC source.

Prelab

DC Circuit

One way of stating Kirchhoff's Current Law (KCL) is that the sum of currents entering a node will equal the sum of currents leaving that node. In the circuit of Figure 1, for example,

$$i_3 = i_1 + i_2$$

One way of stating Kirchhoff's Voltage Law (KVL) is that the algebraic sum of voltage drops around a loop will equal zero. In the circuit of Figure 1, for example, we can consider the loop that passes through the voltage source and resistors R_2 and R_3. We consider moving in a clockwise direction, while treating a voltage drop as positive and a voltage rise as negative.

$$-v_S + v_2 + v_3 = 0$$

This can also be written as

$$v_S = v_2 + v_3$$

This is the statement that the sum of the voltage drops facing across the voltage source must equal the voltage v_S of that source. The voltage across and current through each resistor are related by Ohm's Law:

$$i_1 = \frac{v_2}{R_1}, \quad i_2 = \frac{v_2}{R_2}, \quad i_3 = \frac{v_3}{R_3},$$

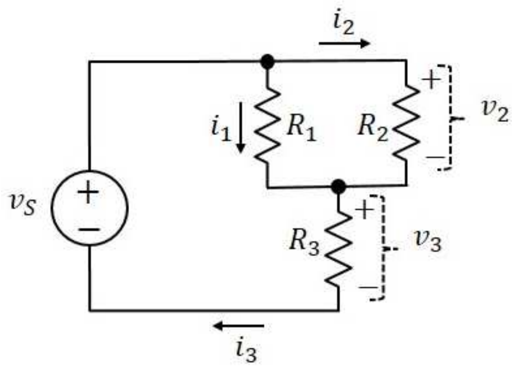

Figure 1: Resistive circuit with DC source

where advantage has been taken of the fact that the voltage across R_1, as well as that across R_2, equals v_2. Combining the above equations permits us to write a system of two linear equations in the unknown variables v_2 and v_3:

$$v_2 + v_3 = v_S$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_2 - \frac{1}{R_3} v_3 = 0$$

Solving this system of two unknown variables in two linear equations, gives the following results:

$$v_2 = \frac{R_1 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} \cdot v_S$$

$$v_3 = \frac{R_1 R_3 + R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \cdot v_S$$

It is noteworthy that both v_1 and v_3 are proportional to v_S. With these results, Ohm's Law can be used to write expressions for i_1, i_2 and i_3. These current variables are also proportional to v_S. If the source voltage v_S were multiplied by a factor K, all the voltages and currents would be multiplied by this same factor K.

The DC power P (W) dissipated in a resistor can be calculated as $P = v \cdot i = v^2/R = i^2 \cdot R$, where v is the voltage (V) across the resistor and i is the current (A) through the resistor. The power delivered by a DC source can be calculated as the voltage of the source times the current supplied by the source.

Exercise: Use KCL and KVL to calculate v_1, v_2, i_1, i_2, and i_3 for the circuit of Figure 2, using the resistor values of Table 2. Calculate the power dissipated in each resistor and the power delivered by the DC source.

AC Circuit

Let's consider the circuit of Figure 1 again, but now the DC voltage source is replaced with an AC voltage source. We would like expressions for the rms voltages across R_2 and R_3 when the rms voltage $V_{S(\text{rms})}$ of the source is known. In solving this problem, we can take advantage of the results already obtained for this circuit with a DC voltage source. It helps to recognize that v_2 and v_3 can be written like this:

$$v_2 = f_2(R_1, R_2, R_3) \cdot v_S$$

$$v_3 = f_3(R_1, R_2, R_3) \cdot v_S$$

v_2, v_3, and v_S represent *instantaneous* voltages when the AC problem is considered. $f_2(\cdot)$ and $f_3(\cdot)$ are functions of the resistances. Applying the formula for the root-mean-square (the square-root of the mean of the square) to both sides of the above equations, we get:

$$V_{2(\text{rms})} = |f_2(R_1, R_2, R_3)| \cdot V_{S(\text{rms})}$$

$$V_{3(\text{rms})} = |f_3(R_1, R_2, R_3)| \cdot V_{S(\text{rms})}$$

In words, the rms voltage $V_{2(\text{rms})}$ across R_2 is proportional to the rms voltage $V_{S(\text{rms})}$ of the source, where the constant of proportionality is the absolute value of the function $f_2(\cdot)$. Similarly, the rms voltage $V_{3(\text{rms})}$ across R_3 is proportional to $V_{S(\text{rms})}$, where the constant of proportionality is the absolute value of the function $f_3(\cdot)$. We note that $V_{S(\text{rms})}$, $V_{2(\text{rms})}$ and $V_{3(\text{rms})}$ are all positive. The rms currents are easily computed from the rms voltages using the rms version of Ohm's Law:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$$

In calculating the rms voltages and currents for an AC circuit, you can use the procedure outlined above, which can be restated as an algorithm:

1. Calculate the rms voltage of the source. If you are given the amplitude of the source, the conversion to rms voltage depends on the waveform. See Table 1.

2. Calculate voltages and currents for the circuit as if the source voltage were DC, where the source voltage equals the rms voltage of the source.

3. After you have completed the calculation of all circuit voltages and currents, take the absolute value of these values. (In other words, if any calculated voltage or current is negative, remove the negative sign.) These are the rms voltages (V_{rms}) and rms currents (I_{rms}).

Table 1: Relationship between X_{rms} and A

Waveform	$\boldsymbol{X_{\mathrm{rms}}}$ **(Exact)**	$\boldsymbol{X_{\mathrm{rms}}}$ **(Approximate)**
Sinewave	$A/\sqrt{2}$	$0.707A$
Square-wave	A	A
Triangle-wave	$A/\sqrt{3}$	$0.577A$

The above procedure for calculating rms voltages and currents in an AC circuit is suitable for a resistive circuit with one AC source. For a circuit containing capacitors or inductors or more than one AC source, a more complicated procedure is required to calculate rms voltages and currents.

The average power P (W) dissipated in a resistor can be calculated as

$$P = \frac{V_{\mathrm{rms}}^2}{R}$$

The average power delivered by a source can be calculated as the rms voltage of the source times the rms current through the source.

Exercise: If the synthesized frequency generator of Figure 7 produces a sinewave with an amplitude of 2 V, find the rms voltage across each resistor and rms current through each resistor in the circuit of Figure 7, using the resistor values of Table 2. Calculate also the power dissipated in each resistor and the power delivered by the AC source.

Procedure

DC Circuit

You will use the three fixed (color coded) resistors listed in Table 2. Measure and record the actual resistance of each resistor that you will use.

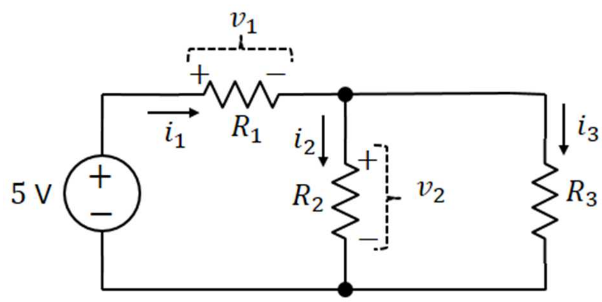

Figure 2: Circuit with DC source

In this experiment, you will use a solderless breadboard and the discrete resistors of Table 2. Construct the circuit of Figure 2, using CH1 of the Siglent power supply. Set the current limit to 0.2 A.

Table 2: Resistors for Figure 2

resistor	value
R_1	3.9 kΩ
R_2	5.6 kΩ
R_3	8.2 kΩ

Measure the voltage across each resistor. To do this, the voltmeter must be placed in parallel with the resistor, as shown in Figure 3. This way, the same voltage appears across the resistor and the voltmeter. We don't want the presence of the voltmeter to affect the voltages and currents in the circuit. With the voltmeter connected in parallel, this means that the voltmeter should look to our circuit like an open circuit. See Figure 4. This ideal is approximated by our voltmeter.

Measure the current through each resistor. To do this, the ammeter must be placed in series with the resistor, as shown in Figure 5. This way, the same current passes through the resistor and the ammeter. We don't want the presence of the ammeter to affect the voltages and currents in the circuit. With the ammeter connected in series, this means that the ammeter should look to our circuit like a short circuit. See Figure 6. This ideal is approximated by our ammeter.

Calculate the power dissipated in each resistor. Calculate the power delivered by the voltage source.

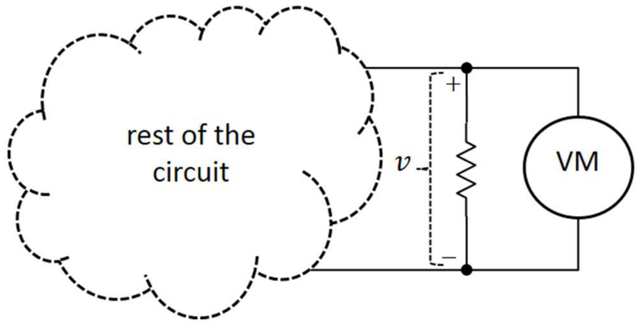

Figure 3: Connecting a voltmeter in parallel

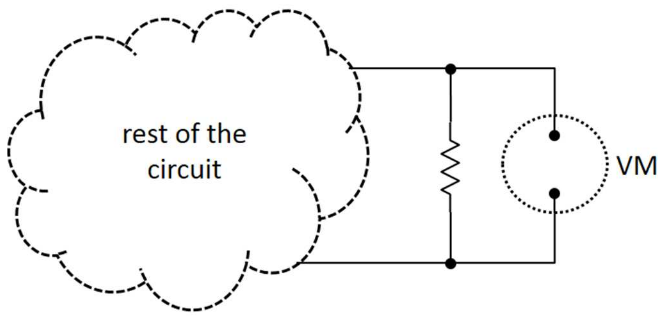

Figure 4: Voltmeter as viewed from the circuit

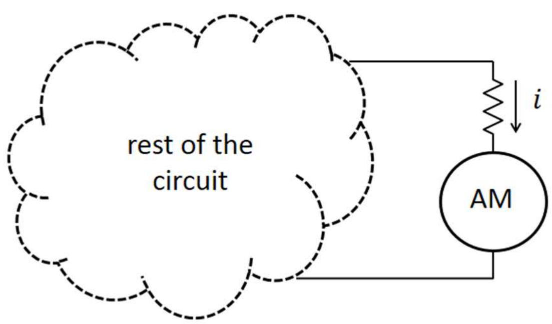

Figure 5: Connecting an ammeter in series

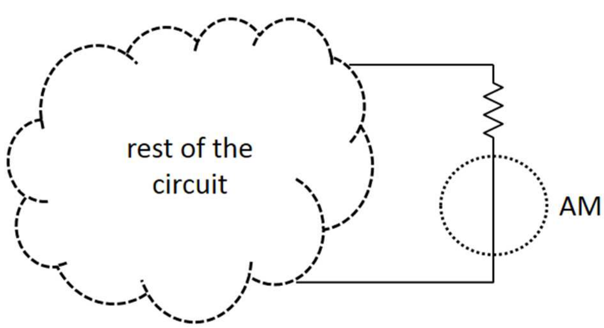

Figure 6: Ammeter as viewed from the circuit

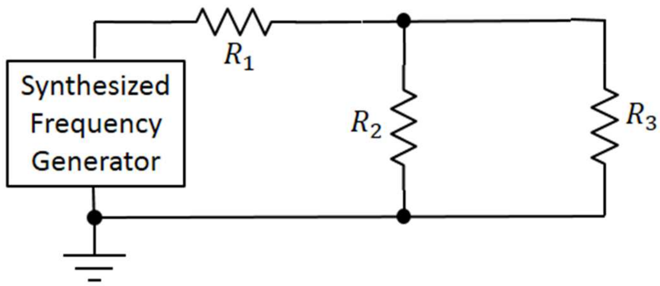

Figure 7: Circuit with AC source

AC Circuit

In the circuit of Figure 2, replace the DC source with the synthesized frequency generator, as shown in Figure 7. Set the AC source to generate a 1-kHz ***sinewave*** with an amplitude of 2.0 V. You should minimize the DC bias of the synthesized frequency generator. You can minimize the DC bias by adjusting the Offset knob of the generator (with the knob in the pulled-out position) while observing the generator output on the oscilloscope (using DC coupling).

We want to measure the rms voltage across R_3 and R_1. The measurement of voltage across R_3 is easily done with the multimeter. An accurate measurement of the voltage across R_1 is more involved, as will be explained below.

Measure the rms voltage across R_3 (which equals, of course, the rms voltage across R_2). Make sure that the voltmeter's COM terminal is connected to the circuit's ground. (For AC measurements with the multimeter, the best accuracy is achieved by having the COM terminal connected to the circuit's ground.)

We cannot measure the voltage across R_1 in this AC circuit using the multimeter without a loss of accuracy. The COM terminal of the multimeter should be at ground for an accurate measurement in an AC circuit, but neither terminal of R_1 is at ground. (We did not face this issue for the DC circuit, since the COM terminal does not need to be at ground when making DC measurements with the multimeter.) Instead, we will use the oscilloscope to make the measurement.

You might think to observe the voltage across R_1 using the connections of Figure 8. Don't. This won't work.

The problem with the connections of Figure 8 is that R_2 gets shorted out. (R_3, which is not shown in Figure 8, is also shorted out.) The outer conductor of the coaxial cable connected to

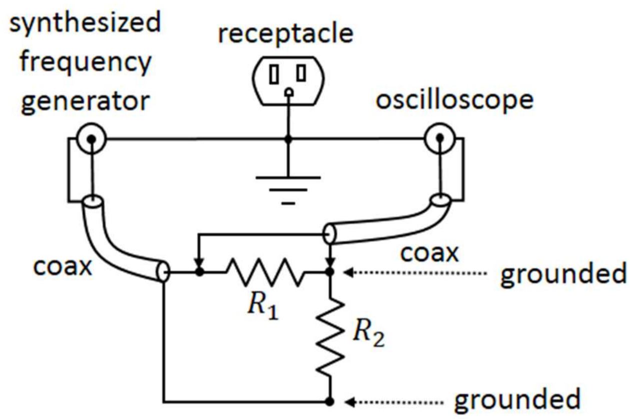

Figure 8: Incorrect connection of oscilloscope

either the oscilloscope's channel-1 or channel-2 input port is grounded inside the instrument. Therefore, when you connect the outer conductor of the coaxial cable (the black alligator clip when using a BNC-to-alligators cable) to a point in the circuit, you are grounding that point. The same is true for the synthesized frequency generator: the outer conductor of the coaxial cable coming from this generator is also grounded.

If you connected the oscilloscope as shown in Figure 8, you would observe a voltage, but it would be the wrong voltage. The resistor R_2 (and also R_3) would be shorted out, and it would be as if only R_1 were connected across the synthesized frequency generator.

Use the following procedure to observe the voltage across R_1. Observe the source voltage on channel 1 and the voltage across R_2 on channel 2. Call the Math menu on the oscilloscope, using the Math button. Ask for a Math channel to be displayed that equals the channel-1 signal minus the channel-2 signal. From the observed Math trace, you should be able to infer the rms voltage across R_1.

Measure the rms current through R_2 and then the rms current through R_3. Make sure that the ammeter's COM terminal is connected to the circuit's ground. (Otherwise, accuracy is lost.)

Measure the rms current through R_1. This can be accomplished while keeping the ammeter's COM terminal connected to the circuit's ground by using the measurement configuration of Figure 9. We can understand that the current measured in this way equals the current i_1 through R_1 by recognizing that the imaginary, dashed box of Figure 9 (containing the parallel combination of R_2 and R_3) must have no net current crossing its boundaries. (The current

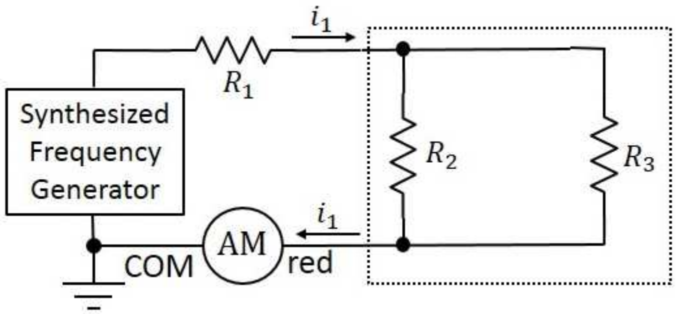

Figure 9: Measuring rms current i_1 through R_1

leaving the box through the lower conductor must equal the current i_1 entering the box on the upper conductor.)

Repeat the above AC voltage and current measurements with the synthesized frequency generator set to produce a 1-kHz ***square-wave*** with an amplitude of 2.0 V.

Repeat the above AC voltage and current measurements with the synthesized frequency generator set to produce a 1-kHz ***triangle-wave*** with an amplitude of 2.0 V.

Lab Report

For the DC circuit, does the measured v_1 plus the measured v_2 equal (at least approximately) 5 V? Does the measured i_2 plus the measured i_3 equal (at least approximately) the measured i_1? Do the sum of the powers dissipated in the three resistors equal (at least approximately) the power delivered by the DC source?

For the AC source, do the rms currents through R_2 and R_3 sum (at least approximately) to the rms current through R_1? Do the sum of the powers dissipated in the three resistors equal (at least approximately) the power delivered by the AC source?