California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 5: Thévenin Equivalent and Maximum Power Transfer

Objective

The objective of this experiment is to demonstrate the Thévenin equivalent of a two-terminal network with DC or AC source.

Prelab

Two-Terminal Network with DC Source

Consider the example two-terminal network of Figure 1. It contains a DC voltage source and three resistors. For a two-terminal network like this, we will often be interested in what happens when external connections are made to the two terminals. By *external* connections, we mean connections not shown in Figure 1. For example, we might be interested in each of the following scenarios: a conductor is connected between the two terminals, or a resistor is connected between the two terminals, or another (different) two-terminal network is connected between the two terminals. If no external connections are made, the current i is zero, of course. But with external connections, this current will, in general, be non-zero.

If we are only interested in the voltage v between the two terminals and the current i flowing through the terminal that has been marked as positive, then we can replace the two-terminal network of Figure 1 with the Thévenin equivalent of Figure 2. Later in this Prelab we explain the procedures for calculating the Thévenin voltage V_{Th} and the Thévenin resistance R_{Th}.

If we replace the original two-terminal network of Figure 1 with the Thévenin equivalent of Figure 2 and then calculate v and i for any set of external connections, we get results that are also correct for the original two-terminal network with the given set of external connections. The advantage of doing the calculations with the Thévenin equivalent rather than with the original two-terminal network is that the Thévenin equivalent is simpler and therefore the calculations are simpler.

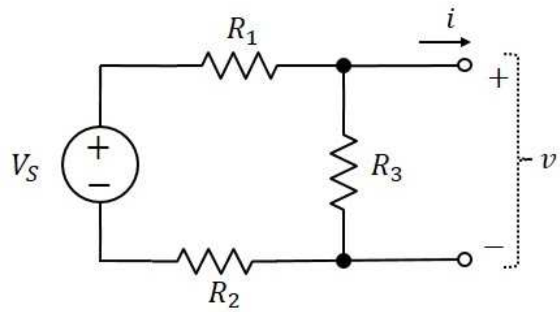


Figure 1: Example two-terminal network with DC source

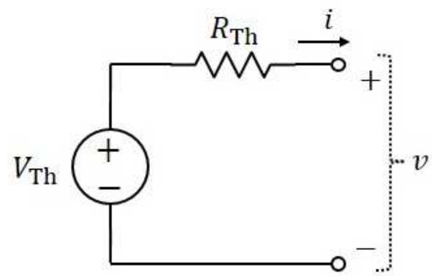


Figure 2: Thévenin equivalent

You might think that there is not much time savings in using the Thévenin equivalent, since this involves a two-step process. First, the Thévenin equivalent must be calculated. Second, the calculations for the Thévenin equivalent with external connections must be carried out. But if you expect to match the two-terminal network with several different sets of external connections, then there can be a large savings in effort due to the fact that the Thévenin equivalent need only be calculated once. That is to say, the first step is only done once and is valid for all sets of external connections. The second step does need to be done for each new set of external connections, but each such calculation is simpler with the Thévenin equivalent than without.

It should be borne in mind that the Thévenin equivalent substitution only helps us with calculating v and i (the voltage and current at the terminals). If we want to know, for example, the current through R_3 in the original two-terminal network of Figure 1, we can't get this directly from the Thévenin equivalent calculations.

Here is the procedure for calculating the Thévenin voltage V_{Th}. We consider the original two-terminal network with no external connections and call this the open-circuit case. (It may be that there is a complete circuit within the original two-terminal network, as there is in Figure 1; but the term "open-circuit" here means that the current i at the terminals is zero because there are no external connections.) We calculate the voltage v under these circumstances and call the result the open-circuit voltage v_{oc}. The Thévenin voltage is:

$$V_{\text{Th}} = v_{\text{oc}},$$

where v_{oc} = open-circuit voltage.

From Figure 2 we see that the open-circuit voltage for the Thévenin equivalent is also V_{Th}. Later when we make an external connection between the two terminals, we continue to use the same numeric value for the Thévenin source V_{Th}; but, of course, the voltage v between the two terminals will change.

Here is the procedure for calculating the Thévenin resistance R_{Th}. We consider the original two-terminal network with a simple conductor (a short circuit) connecting the two terminals and call this the closed-circuit case. We calculate the current i under these circumstances and call the result the short-circuit current i_{sc}. The Thévenin resistance is:

$$R_{\text{Th}} = \frac{V_{\text{Th}}}{i_{\text{sc}}},$$

where i_{sc} = short-circuit current.

From Figure 2 we see that the short-circuit current for the Thévenin equivalent is $V_{\text{Th}}/R_{\text{Th}}$.

When the Thévenin voltage and resistance are selected as described above, the original two-terminal network and the Thévenin equivalent are in agreement about the terminal voltage and current under both open-circuit and short-circuit conditions. Although we don't prove it here, it is generally true that *the original two-terminal network and the Thévenin equivalent are in agreement about the terminal voltage and current for all external connections*, not just the special open-circuit and short-circuit cases.

In the example two-terminal network of Figure 1, we consider the open-circuit case and recognize that V_{Th} is given by a voltage division for the two-terminal network in Figure 1:

$$V_{\text{Th}} = \frac{R_3}{R_1 + R_2 + R_3} \cdot V_S$$

Also for the two-terminal network in Figure 1, we consider the short-circuit case and recognize that i_{sc} is given by:

$$i_{\text{sc}} = \frac{V_S}{R_1 + R_2}$$

and therefore that:

$$R_{\text{Th}} = \frac{(R_1 + R_2) \cdot R_3}{R_1 + R_2 + R_3}$$

Exercise: Find the Thévenin equivalent for the two-terminal network with DC source of Figure 3, where the resistors have the values given in Table 1. You will use the procedures described above for calculating V_{Th} and R_{Th}. (Please note that the equations given above that are labeled "for the two-terminal network of Figure 1" are not applicable in this problem.)

Two-Terminal Network with AC Source

We can also find a Thévenin equivalent for a two-terminal network having an AC voltage source. The Thévenin equivalent looks like Figure 2, except that the Thévenin voltage source is an AC source that produces the same waveform as the original two-terminal network. If the original two-terminal network contains a sinewave source, then the Thévenin equivalent contains a sinewave source. If the original two-terminal network contains a square-wave source, then the Thévenin equivalent contains a square-wave source. If the original two-terminal network contains a triangle-wave source, then the Thévenin equivalent contains a triangle-wave source.

Here we use V_{Th} to represent the *amplitude* of the Thévenin equivalent's source. V_{Th} is found as the *amplitude* of the terminal voltage v when there are no external connections. That is to say, V_{Th} is the *amplitude* of the open-circuit voltage v_{oc}.

R_{Th} is found as the ratio of V_{Th} to the *amplitude* of the short-circuit current i_{sc}.

Exercise: Consider the two-terminal network of Figure 3, but with the DC voltage source replaced with a sinewave voltage source having an amplitude of 5 V. The resistors have the values given in Table 1. Find the Thévenin equivalent.

Procedure

You will use decade boxes for the resistors in this experiment. For each resistor, select a decade box. Set the correct resistance for each decade box in accord with Table 1. Measure and record the actual resistance for each.

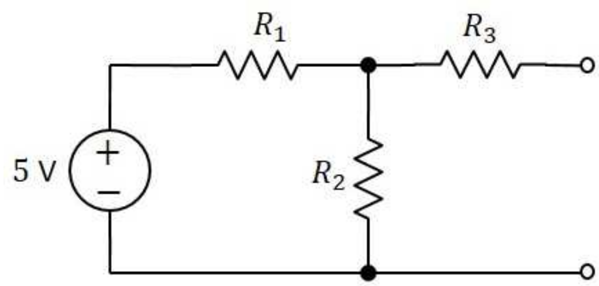


Figure 3: Two-terminal network with DC source

Table 1: Resistors for Figures 3, 4, and 6

Resistor	Value
R_1	8 kΩ
R_2	8 kΩ
R_3	6 kΩ

Two-Terminal Network with DC Source

Construct the two-terminal network with DC source that is shown in Figure 3 with the resistor values of Table 1. Use CH1 of the Siglent power supply. Set the current limit to 0.2 A.

Place the voltmeter between the two open terminals and measure the open-circuit voltage. Replace the voltmeter with the ammeter and measure the short-circuit current. From the open-circuit voltage and short-circuit current, calculate the Thévenin voltage and Thévenin resistance. These values should be close to those values that you calculated in the Prelab.

Construct the Thévenin-equivalent network, as shown in Figure 2, where V_{Th} and R_{Th} are the Thévenin voltage and Thévenin resistance as determined above. Use CH1 of the Siglent power supply for V_{Th}, and set its current limit to 0.2 A.

Measure the open-circuit voltage and short-circuit current for the Thévenin-equivalent network of Figure 2. Each of these values should be close to the corresponding value measured for the original network (Figure 3).

You will use a fourth resistor decade box as a load resistor. For each value of load resistance, set the resistor decade box to that value and then measure the actual resistance. The values of load resistance that we will use are: 5 kΩ, 10 kΩ, 15 kΩ, and 20 kΩ.

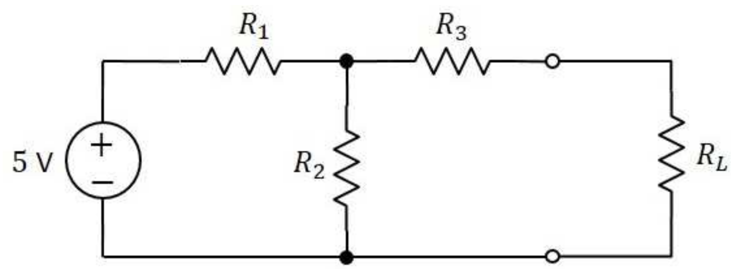


Figure 4: Two-terminal network with load

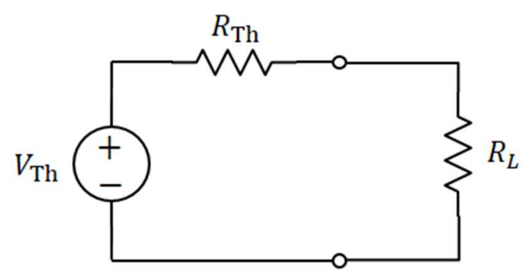


Figure 5: Thévenin Equivalent with load

Add the load resistor to the two-terminal network of Figure 3, as indicated in Figure 4. For each value of load resistance, measure the voltage v_L across the load and the current i_L through the load. From these measurements, calculate the power p_L delivered to the load.

Apply the same set of loads to the Thévenin-equivalent network, as shown in Figure 5. For each load, measure v_L and i_L, then calculate p_L.

Two-Terminal Network with AC Source

In the two-terminal network of Figure 3, replace the DC source with the synthesized frequency generator, as shown in Figure 6. Set the generator to produce a 1-kHz sinewave with an amplitude of 5 V.

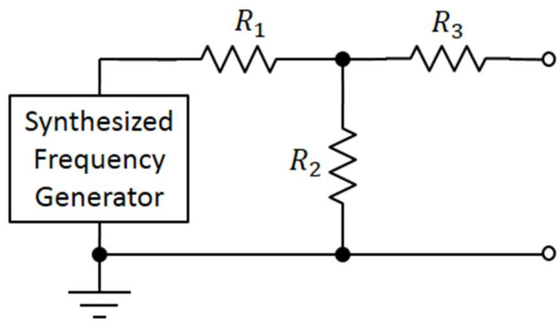


Figure 6: Two-terminal network with AC source

Use the AC voltmeter to measure the RMS value of the open-circuit voltage v_{oc}. Use the AC ammeter to measure the RMS value of the short-circuit current i_{sc}. Convert each of these RMS values to amplitudes. Table 2 is a reminder of how the RMS value, denoted X_{rms}, of an AC signal is related to the amplitude A of that signal. (In this experiment, you will only be using one row from this table.) V_{Th} is the amplitude of the open-circuit voltage. R_{Th} is the ratio of V_{Th} to the amplitude of the short-circuit current. (Actually, R_{Th} can also be calculated as the ratio of the RMS open-circuit voltage to the RMS of the short-circuit current.)

Table 2: Relationship between X_{rms} and A

Waveform	**X_{rms} (Exact)**	**X_{rms} (Approximate)**
sinewave	$A/\sqrt{2}$	$0.707A$
square-wave	A	A
triangle-wave	$A/\sqrt{3}$	$0.577A$

Add the load resistor to the two-terminal network of Figure 6. The synthesized frequency generator will remain set for a 1-kHz sinewave with an amplitude of 5 V. For each value of load resistance, measure the RMS voltage V_L across the load and the RMS current I_L through the load. From these measurements, calculate the average power P_L delivered to the load.

Construct the Thévenin-equivalent network, as shown in Figure 7, where R_{Th} is the Thévenin resistance as determined above. The synthesized frequency generator acts here as the Thévenin source; its amplitude should equal the amplitude of the open-circuit voltage that you calculated earlier (from a measurement of the RMS open-circuit voltage).

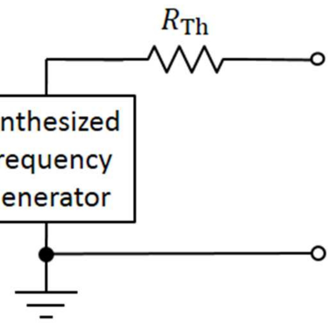
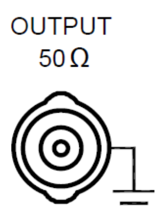


Figure 7: Thévenin equivalent for network of Figure 6

Apply the same set of loads to the Thévenin-equivalent network. For each value of load resistance, measure the RMS voltage V_L across the load and the RMS current I_L through the load. From these measurements, calculate the average power P_L delivered to the load.

Synthesized Frequency Generator as a Two-Terminal Network

We may regard the synthesized frequency generator as a two-terminal network with AC source. You may have noticed that the output BNC port of this generator is labeled "50 Ω". This means that the Thévenin resistance of the synthesized frequency generator, as viewed through its output port (consisting of the two terminals: the BNC bayonet and the BNC connector body), is 50 Ω.



Set a resistor decade box to 50 Ω. Measure its actual resistance. Place the synthesized frequency generator output across this resistor. The load should therefore match the Thévenin resistance. Set the synthesized frequency generator to produce a 1-kHz sinewave. For the amplitudes listed in Table 3, measure the RMS voltage across the load:

Table 3: Thévenin RMS and Load RMS voltage Measurements

Thévenin Amplitude	Thévenin RMS Voltage	Load RMS Voltage
2 V		
4 V		
6 V		

The Thévenin amplitude is the amplitude indicated by the synthesized frequency generator. The second column is the corresponding RMS voltage. The third column is the RMS voltage measured across the load.

Most of the time when we use the synthesized frequency generator, we load it with a much larger resistance than 50 Ω, so almost all of the Thévenin amplitude appears across the load. In such a case we typically call the Thévenin amplitude of this generator simply the "amplitude". But, as illustrated above, when the load is small, we ought to account for the Thévenin resistance of this generator and the resulting voltage division.

Finally, there is a matter of vocabulary. We don't normally call the synthesized frequency generator's 50 Ω the "Thévenin resistance". (Although we can call it that. It is an accurate description.) Instead, we normally call it the "output impedance". Impedance is a generalization of the resistance concept. When an impedance is purely resistive in character, then the output impedance is really just an output resistance. Even so, the more general term "output impedance" is preferred.

Lab Report

Does your experimental determination of V_{Th} and R_{Th} agree well with your Prelab calculations?

For the Thévenin equivalent circuit with load of Figure 5, plot p_L (the power delivered to the load) as a function of R_L. This figure should contain both a solid curve for the theory and discrete points for your measured data.
Is the power delivered to the load maximized by a load that matches R_{Th}?

When you place a 50 Ω load across the synthesized frequency generator, do you measure a load voltage that meets with your expectations?