California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 6: Superposition

Objective

The objective of this experiment is to demonstrate the superposition principle. This is done for circuits having both DC and AC voltage sources.

Prelab

For a linear circuit with more than one voltage source, the superposition principle offers us an alternative method to calculating voltages and currents.

DC Circuits

Consider the circuit of Figure 1. It contains two DC voltage sources. We could use KVL and KCL to write a set of three linear equations in the three unknown currents (one current for each resistor). This set of linear equations could be solved for the three currents. Then from each current we could calculate the voltage across the corresponding resistor.
If we are only interested in one voltage (or one current), however, it is often easier to use the superposition principle. The calculation of v_3 using superposition is done below. But first the method of superposition is described.

The superposition method typically simplifies our calculations for a linear circuit containing more than one source. In the superposition method, we set all but one voltage source to zero and then calculate the contribution of the one voltage source (the one that has not been set to zero) to the desired voltage (or current). We calculate the contribution of each voltage source to the desired voltage (or current) in this way. After we have a contribution from each of the voltage sources, we add those contributions together.

For the circuit of Figure 1, we want to know the voltage v_3. With the superposition method, we first calculate the contribution of the source V_{S1} to v_3 by setting V_{S2} to zero. Setting a voltage source to zero is achieved by replacing that source with a simple conductor (a short), because there is zero voltage from one end of a good conductor to the other. Figure 2 illustrates the situation.

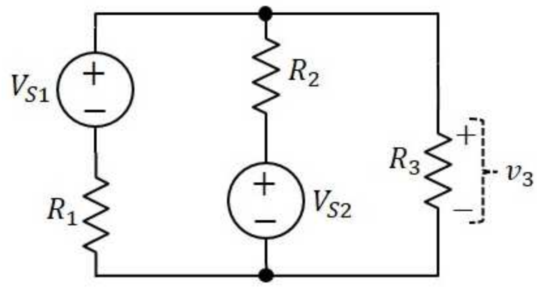

Figure 1: Example circuit with two DC voltage sources

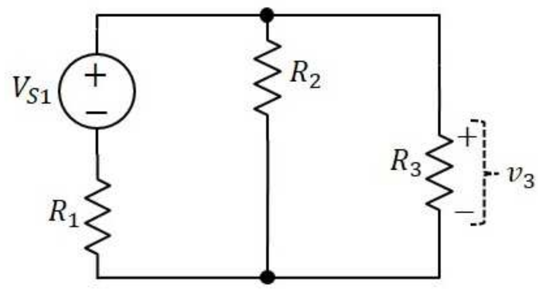

Figure 2: V_{S2} of Figure 1 has been set to 0

We can calculate the contribution of V_{S1} to v_3 by examining Figure 2. For the modified circuit of Figure 2, v_3 is the voltage across the parallel combination of R_2 and R_3. The contribution of V_{S1} to v_3 is given by the voltage division:

$$\frac{\left(\frac{R_2R_3}{R_2 + R_3}\right)}{R_1 + \left(\frac{R_2R_3}{R_2 + R_3}\right)} \cdot V_{S1} = \frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} \cdot V_{S1}$$

Next, we calculate the contribution of the source V_{S2} to v_3 by setting V_{S1} to zero. Figure 3 illustrates this situation.

We can calculate the contribution of V_{S2} to v_3 by examining Figure 3. For the modified circuit of Figure 3, v_3 is the voltage across the parallel combination of R_1 and R_3. The contribution of V_{S2} to v_3 is given by the voltage division:

$$\frac{\left(\frac{R_1R_3}{R_1 + R_3}\right)}{R_2 + \left(\frac{R_1R_3}{R_1 + R_3}\right)} \cdot V_{S2} = \frac{R_1R_3}{R_1R_2 + R_1R_3 + R_2R_3} \cdot V_{S2}$$

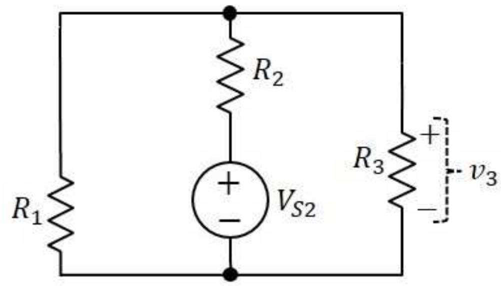

Figure 3: V_{S1} of Figure 1 has been set to 0

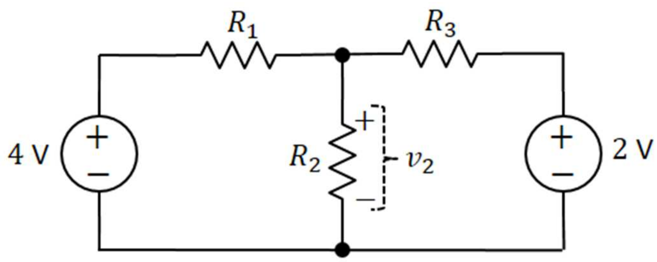

Figure 4: Circuit with two DC voltage sources

Finally, accounting for the contributions from both V_{S1} and V_{S2}, the actual voltage v_3 is:

$$v_3 = \frac{R_2 R_3 \cdot V_{S1} + R_1 R_3 \cdot V_{S2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

for the circuit of Figure 1.

Exercise: For the circuit of Figure 4, using the resistor values of Table 1, calculate v_2 (the voltage across R_2). You should use the principle of superposition. (Please note that the equation given above that is labeled "for the circuit of Figure 1" is not applicable in this problem.)

Table 1: Resistors for Figure 4

Resistor	Value
R_1	30 kΩ
R_2	20 kΩ
R_3	10 kΩ

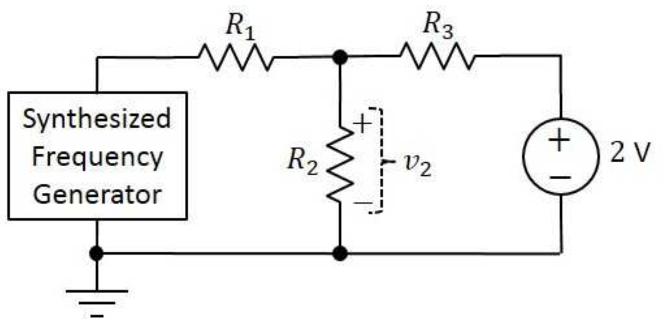

Figure 5: Circuit with one AC and one DC voltage source

AC Circuits

Consider the circuit of Figure 1 again, but now let's see what happens when the DC voltage source V_{S1} is replaced by an AC voltage source v_{AC}. We can use the principle of superposition here as well, and the result is:

$$v_3 = \frac{R_2 R_3 \cdot v_{AC}}{R_1 R_2 + R_1 R_3 + R_2 R_3} + \frac{R_1 R_3 \cdot V_{S2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

The first term of the equation is an AC voltage, which is the contribution of v_{AC} to v_3. The second term is a DC voltage, which is the contribution of V_{S2} to v_3. The peak (that is, maximum value) of v_3 is given by

$$v_{3\text{ peak}} = \frac{R_2 R_3 \cdot (v_{AC\text{ peak}})}{R_1 R_2 + R_1 R_3 + R_2 R_3} + \frac{R_1 R_3 \cdot |V_{S2}|}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Exercise: In the circuit of Figure 5, the AC voltage source has an amplitude (peak) of 2.0 V and the resistors have the values given in Table 1. Calculate the peak of v_2 using the principle of superposition. (Please note that the equations given above that are labeled "for the circuit of Figure 1" are not applicable in this problem.)

Figure 6 is another example of a circuit having both an AC and a DC voltage source. This circuit also contains a capacitor, in addition to resistors.

A capacitor looks like an open circuit to DC, meaning no constant (DC) current passes through a capacitor, but a constant, non-zero voltage can appear across a capacitor. An AC current, on the other hand, can pass through a capacitor. We will model the capacitor of Figure 6 as an open circuit to DC, and a short circuit to AC, and use superposition to calculate v_3. In general, one must treat a capacitor as having an impedance to the flow of an AC current. However, in the

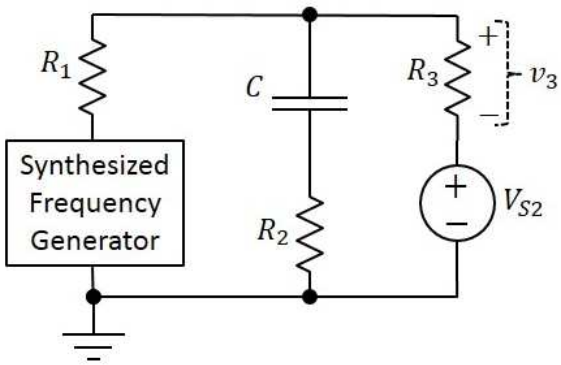

Figure 6: Superposition for circuit with capacitor and resistors

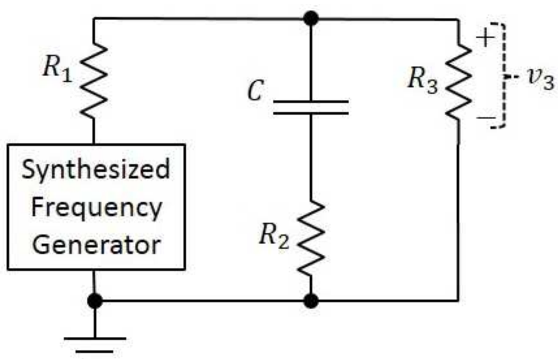

Figure 7: V_{S2} of Figure 6 has been set to 0

circuit of Figure 6, we will assume that the impedance of the capacitor to the flow of an AC current is negligibly small. Our result will only be approximate, though, since we are modeling the impedance of the capacitor as zero, and in reality the impedance may be small to AC, but it will be non-zero.

First, we imagine the DC source being set to zero. The resulting, simplified circuit is shown in Figure 7. Here we are modeling the capacitor as a short circuit for an AC signal. Therefore, the resistors R_2 and R_3 are essentially in parallel. Denoting by v_{AC} the voltage of the AC source, we find that the contribution of v_{AC} to v_3 is:

$$\frac{R_2R_3 \cdot v_{AC}}{R_1R_2 + R_1R_3 + R_2R_3}$$

Second, we can return to Figure 6 (with the DC source and capacitor back in place) and then imagine the AC source being set to zero. The resulting, simplified circuit is shown in Figure 8. Since a capacitor blocks a DC current, we have replaced the capacitor in this diagram with an open circuit. We find that the contribution of V_{S2} to v_3 is:

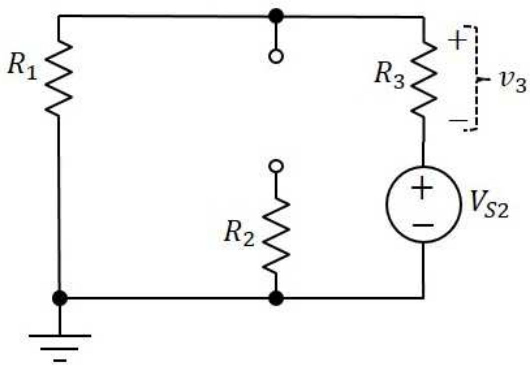

Figure 8: AC source of Figure 1 has been set to 0

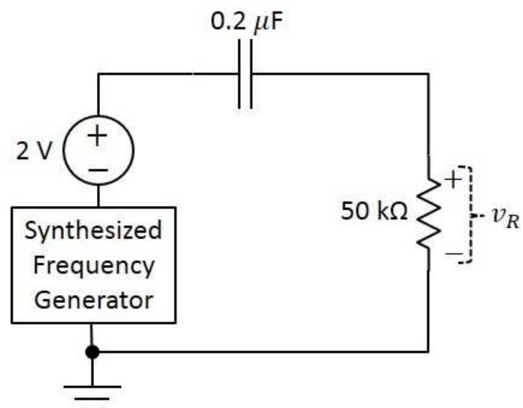

Figure 9: Circuit with a DC and AC voltage sources, capacitor, and resistor

$$-\frac{R_3}{R_1 + R_3} \cdot V_{S2}$$

The negative sign is due to the fact that the "bottom" terminal of R_3 is connected to the positive terminal of V_{S2}. In other words, if V_{S2} is positive, the contribution of V_{S2} to v_3 will be negative.

Adding the two contributions to v_3, we get:

$$v_3 = \frac{R_2 R_3 \cdot v_{AC}}{R_1 R_2 + R_1 R_3 + R_2 R_3} - \frac{R_3}{R_1 + R_3} \cdot V_{S2}$$

Exercise: Consider circuit of Figure 9. If the synthesized frequency generator produces a 2-kHz sinewave with amplitude 2.0 V, identify the DC component of v_R and the amplitude of the AC component of v_R. You may assume that the capacitor has negligible impedance to an AC current, and can be modeled as a short circuit. The capacitor can always be modeled as an open circuit to a DC signal.

Procedure

Circuit of Figure 4

You will build the circuit of Figure 4 using the resistances of Table 1. Use decade boxes for the resistors. Measure and record the actual resistance for each resistor.

Use CH1 and CH2 of the Siglent power supply for the 4-V and 2-V DC voltage sources. Set the current limit to 0.2 A for each. Measure v_2.

Circuit of Figure 5

In the circuit of Figure 4, replace the 4-V DC source with the synthesized frequency generator, as shown in Figure 5. Set the AC source to generate, initially, a 2-kHz ***sinewave*** with an amplitude of 2.0 V.

For the measurements with this circuit, it is important to know exactly were 0 V is on the oscilloscope display for each channel. It will be convenient if 0 V coincides with the horizontal axis (the horizontal line that is vertically centered in the display). Make sure that the horizontal axis of the oscilloscope corresponds to 0 V for each oscilloscope channel. You can do this, for each channel, by changing the coupling to ground, so that 0 V is displayed, and then, if necessary, adjusting the vertical position of the 0-V trace so that is coincides with the horizontal axis. After doing this, change the coupling to DC.

It is important to use DC coupling on the oscilloscope, rather than AC coupling, in this experiment. DC coupling means that both DC and AC components are displayed. (That is to say, *everything* is displayed.) AC coupling, on the other hand, means that DC components are blocked, so that *only* AC components are displayed.

You should also minimize the DC bias of the synthesized frequency generator. You can minimize the DC bias by adjusting the Offset knob of the generator (with the knob in the pulled-out position) while observing the generator output on the oscilloscope (using DC coupling).

Observe v_2 on the oscilloscope using DC coupling. Make a note of the maximum value of v_2.

Change the AC source (the synthesized frequency generator) to a 2-kHz ***square-wave*** with amplitude 2.0 V. Observe v_2 on the oscilloscope. Use DC coupling. Make a note of the maximum value of v_2.

Change the AC source to a 2-kHz ***triangle-wave*** with amplitude 2.0 V. Observe v_2 on the oscilloscope. Use DC coupling. Make a note of the maximum value of v_2.

Circuit of Figure 9

You will build the circuit of Figure 9. Select a resistive decade box for the 50-kΩ resistor, set the correct resistance, and measure it. Select a capacitive decade box for the 0.2-μF capacitor, and set the correct capacitance.

Use CH1 of the Siglent power supply for the 2-V DC voltage source. It is essential that the CH1 − terminal *not* be connected to ground. Set the current limit to 0.2 A.

Set the AC source to generate, initially, a 2-kHz ***sinewave*** with an amplitude of 2.0 V. You should minimize the DC bias using the Offset knob (pulled out) of the synthesized frequency generator.

Make sure that the horizontal axis of the oscilloscope corresponds to 0 V for both channels.

On channel 1, display the voltage at the positive terminal of the DC source, relative to ground. (This voltage will be the sum of the AC source voltage and the DC source voltage.) On channel 2, display v_R (the voltage across the 50-kΩ resistor). For both channels use DC coupling.

Measure the DC component for each channel. On the oscilloscope's Measure menu, you will select Mean to get a measurement of the DC component.

Measure the peak-to-peak value of the AC component for each channel. On the oscilloscope's Measure menu, you will select Pk-Pk to get a measurement of the peak-to-peak value of the AC component.

Change the AC source to a 2-kHz ***square-wave*** with amplitude 2.0 V and repeat the above measurements.

Change the AC source to a 2-kHz ***triangle-wave*** with amplitude 2.0 V and repeat the above measurements.

Lab Report

For the circuit of Figure 4, did the measured value of v_2 meet with your expectation?

For the circuit of Figure 5, did the measured maximum of v_2 meet with your expectation?

For the circuit of Figure 9, does the principle of superposition predict (at least approximately) the voltage that you observed across the 50-kΩ resistor?