California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 11: Operational Amplifier Integrator and Differentiator

Objective

The objective of this experiment is to study the behavior of two op-amp circuits: one that approximates an integrator and a second that approximates a differentiator.

Prelab

Ideal Integrator

An ideal integrator is shown in Figure 1. At first glance, it might look like an inverting amplifier. The input path has a resistor R_i. There is negative feedback present. But the feedback path, which connects the output terminal to the inverting input terminal of the op amp, has a capacitor C_f, instead of a resistor.

We can understand how this circuit functions as an integrator with the following logic. Because this circuit has negative feedback,

$$i_i + i_0 = 0$$

Since the inverting input terminal of the op amp is a virtual ground:

$$i_i = \frac{v_i}{R_i}$$

and

$$i_0 = C_f \frac{dv_0}{dt}$$

Combining these three equations gives

$$C_f \frac{dv_0}{dt} = -\frac{v_i}{R_i}$$

or, equivalently,

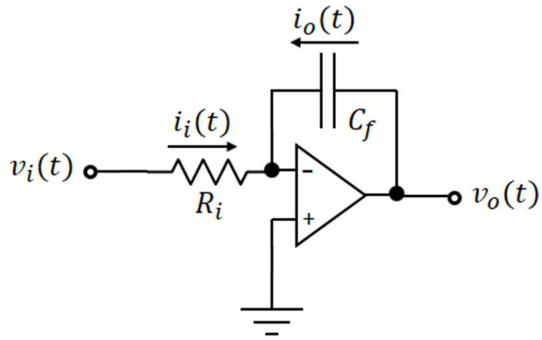

Figure 1: Ideal integrator

$$v_0 = -\frac{1}{R_i C_f} \int_0^t v_i(t')\, dt'$$

In words, the output voltage equals the integral of the input voltage, times $\left(-1/(R_i C_f)\right)$.

We call this an *ideal* integrator because it only works this simply in our imagination. In practice, we can only approximate an integrator, and the approximation requires us to add components to the relatively simple circuit of Figure 1. More is said about this later.

Exercise: If $v_i(t)$ is a square-wave, what is the wave shape of $v_0(t)$? If this square-wave $v_i(t)$ has an amplitude of 2 V and a frequency of 1 kHz, R_i is 75 kΩ and C_f is 0.0022 μF, what is the amplitude of $v_0(t)$?

Exercise: For $v_i(t) = 5\cos(2\pi f t)$ with $f = 1$ kHz, write an expression for $v_o(t)$. If R_i is 75 kΩ and C_f is 0.0022 μF, what is the amplitude of $v_o(t)$?

Ideal Differentiator

An ideal differentiator is shown in Figure 2. At first glance, it may look like an inverting amplifier. There is negative feedback present. The feedback path, between the output terminal and the inverting input terminal of the op amp, has a resistor R_f. But the input path has a capacitor C_i, instead of a resistor.

We can understand how this circuit functions as a differentiator with the following logic. Because this circuit has negative feedback,

$$i_i + i_0 = 0$$

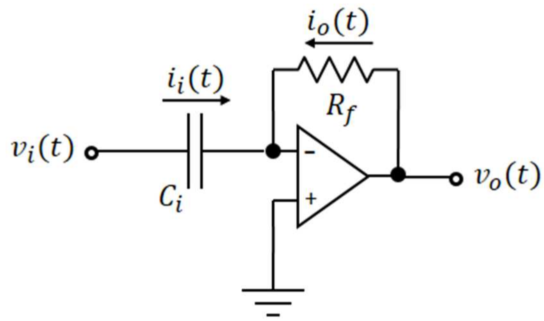

Figure 2: Ideal differentiator

Since the inverting input terminal of the op amp is a virtual ground,

$$i_i = C_i \frac{dv_i}{dt}$$

and

$$i_0 = \frac{v_0}{R_f}$$

Combining these three equations gives

$$\frac{v_o}{R_f} = -C_i \frac{dv_i}{dt}$$

or, equivalently,

$$v_0 = -R_f C_i \frac{dv_i}{dt}$$

In words, the output voltage equals the derivative of the input voltage, times $\left(-R_f C_i\right)$.

We call this an *ideal* differentiator because it doesn't work well in practice if we don't add at least one other component. More is said about this later.

Exercise: If $v_i(t)$ is a triangle-wave, what is the wave shape of $v_0(t)$? If this triangle-wave $v_i(t)$ has an amplitude of 2 V and a frequency of 1 kHz, R_i is 75 kΩ and C_f is 0.0022 μF, what is the amplitude of $v_0(t)$?

Exercise: For $v_i(t) = 2\sin(2\pi f t)$ with $f = 1$ kHz, write an expression for $v_o(t)$. If R_f is 75 kΩ and C_i is 0.0022 μF, what is the amplitude of $v_o(t)$?

Procedure

For reasons that are mentioned below, the circuits of Figure 1 and 2 are not practical. You will build a practical integrator and a practical differentiator that are approximations to the ideal circuits you considered in the Prelab.

Place an LM741 op amp on the solderless breadboard. The pin-out of this op amp is illustrated in Figure 3.

Configure the Siglent DC power supply to be the source of $+12$ V for pin 7 and -12 V for pin 4. But do not apply live voltages to the op amp until ready to make measurements. Set the maximum current to 0.2 A for both sources. Place the DC power supply in series mode. Set the CH1 voltage to 12 V, causing the CH 2 voltage to be 12 V also. Connect the CH2 $-$ terminal, which is internally connected to the CH1 $+$ terminal while in series mode, to ground. With this configuration, the CH2 $+$ terminal is at $+12$ V relative to ground and the CH1 $-$ terminal is at -12 V relative to ground.

Integrator

Consider the integral of a constant A:

$$\int_0^t A\, dt' = A\, t$$

The result of this integration keeps growing if $A > 0$ or keeps decreasing if $A < 0$. For an op-amp integrator, any DC component on the input, even a small one, is a problem. If there is any DC component, the circuit output will ramp until it reaches a rail. (Recall that an op-amp integrator introduces a negative sign. A negative DC component at the input will cause the output to reach the positive rail. A positive DC component at the input will cause the output to reach the negative rail.) An "integrator" whose output sits at one of the rails is not doing its job. There are two actions we must take to ensure that our op-amp integrator doesn't reach a rail.

First, we will couple the source of the input signal to the integrator circuit through a capacitor. See Figure 4. This capacitor will look like an open circuit to a DC component. The capacitor should be big enough that it does not significantly impede the AC frequencies that we intend to use with this circuit. We will use a 0.2-μF capacitor, which offers little impedance to frequencies of 1 kHz or larger. With this first action we prevent any DC component originating

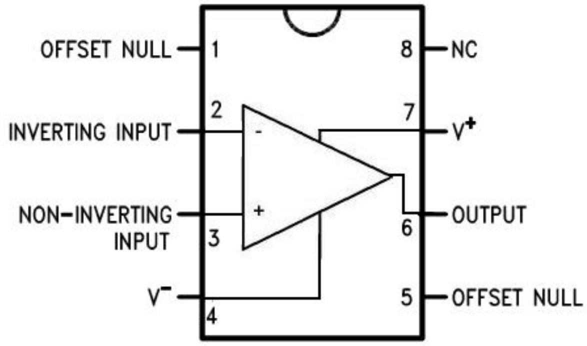

Figure 3: LM741 pin-out

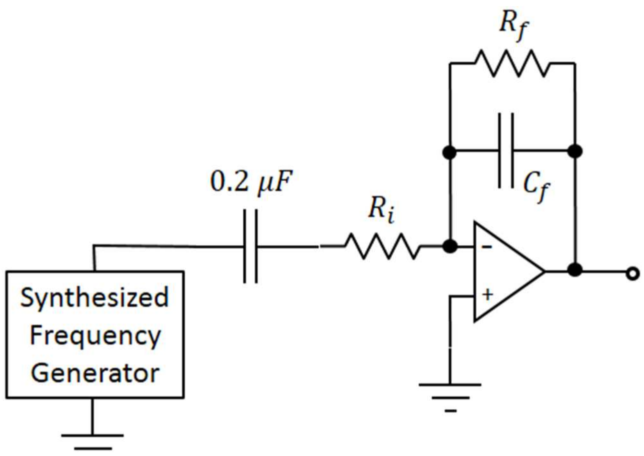

Figure 4: Practical integrator (with addition of feedback resistor R_f), receiving input through a capacitor

in the synthesized frequency generator from reaching the input of the integrator circuit. (It is good practice to minimize the DC bias in the sinewave using the Offset knob on the synthesized frequency generator before applying the sinewave to the circuit, but this minimization of the DC bias does not completely eliminate it.)

Second, we add a resistor R_f in parallel to the capacitor C_f that is in the feedback path of our integrator circuit. The feedback path now looks like the resistor R_f alone to a DC component, since the capacitor C_f looks like an open circuit to DC. Therefore, the circuit looks like an inverting amplifier, rather than an integrator, to a DC component. An amplifier will, of course, amplify a DC component at its input. (But this is much better than having a ramping output that ends at a rail.) If the DC component at the input is very small and if the gain is not too large, then the DC bias on the output might still be small.

Given that the 0.2-μF capacitor of Figure 4 blocks any DC component originating in the generator from reaching the integrator input, why are we worried about a DC component on the integrator input? The answer is that in practical op amps there are very small DC currents entering the inverting and noninverting inputs of the op amp. The DC current on each of the inverting and non-inverting inputs of the LM741 is typically less than 100 nA (that is, 10^{-7} A). In most op-amp circuits, these tiny DC currents can be ignored, and we analyze those circuits as if op-amp input currents were zero. But for an integrator, even a very small DC current or voltage at its input can be a problem (because DC at the input causes the output of an integrator to ramp to a rail). We have already taken the action that solves this problem. The additional feedback resistor R_f makes the circuit look like an inverting amplifier (rather than an integrator) to DC.

We hope that the op-amp circuit will look like an integrator to an AC signal, even with the feedback resistor R_f in place. (After all, the purpose of this exercise is to build a circuit that integrates AC signals.) To address this issue, we must consider the actual component values. Table 1 shows the component values to be used for the practical integrator of Figure 4.

Table 1: Component values for the practical integrator of Figure 4

Component	Integrator
R_i	75 kΩ
C_f	0.0022 μF
R_f	750 kΩ

For the component values we will be using, the circuit of Figure 4 approximates an integrator when the input frequency is much larger than 100 Hz. When $f \gg 100$ Hz, C_f has an impedance whose magnitude is small compared with R_f. The character of the feedback path is, in this case, determined by C_f. (With two impedances in parallel, the impedance with the smaller magnitude dominates.)

Build the circuit of Figure 4 using the component values of Table 1.

Temporarily remove R_f from the circuit of Figure 4. With any signal as input (a sinewave from the synthesized frequency generator, for example), you should find that the integrator output sits on one of the rails. (Actually, the output will have *ramped* to the rail, but the ramping happened so fast that you didn't see it.) This confirms that the ideal integrator doesn't work in practice, due to the DC bias currents on the op-amp inputs.

Restore R_f to your circuit. Use the synthesized frequency generator to create a sinewave of amplitude 5 V. Adjust the offset to minimize the DC bias on the generator output. Set the frequency f to each of the following values: 1 kHz, 2 kHz, and 4 kHz. For each frequency, observe the input and output signals of the (practical) integrator on channels 1 and 2 of the oscilloscope. Estimate the phase of the output sinewave relative to that of the input sinewave (that is, the output phase minus the input phase).

Make a table that shows, for each frequency, the input amplitude, the predicted output amplitude, the measured output amplitude, the predicted phase difference, and the observed phase difference. For the predicted amplitude and the predicted phase difference, ignore R_f. (That is to say, the calculations that lead to your predictions should be based on an *ideal* integrator.)

Use the synthesized frequency generator to create a *square-wave* (instead of a sinewave) of amplitude 2 V. Set the frequency first to 1 kHz and then to 2 kHz. Observe the input and output signals on channels 1 and 2 of the oscilloscope.

Differentiator

For an ideal differentiator, the output signal is the derivative of the input signal. This is a useful operation in analog signal processing.

Unfortunately, the ideal differentiator has one property that leads to practical difficulties. The derivative of $\sin(2\pi f t)$ is $2\pi f \cdot \cos(2\pi f t)$. The output signal has an amplitude that is proportional to the frequency. You might think that you will not run into problems as long as you keep the signal frequency at the input relatively small, so of that the amplitude of the output voltage remains within the rails. In practice, however, we *do* have a problem, even if we constrain the input signal frequency. In practical circuits, there is always noise present. Noise can be modeled as a sum of sinewaves with a broad range of frequencies, each sinewave having a random amplitude and phase. The noise at the input of an ideal differentiator will include high-frequency sinewaves that, though initially small in amplitude, get greatly amplified by an ideal differentiator.

The noise problem of an ideal differentiator can be solved with the practical differentiator circuit shown in Figure 5. To understand how this practical differentiator works, we must consider the actual component values. Table 2 shows the component values to be used for the practical differentiator of Figure 5.

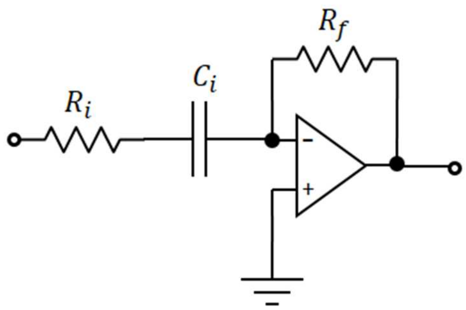

Figure 5: Practical differentiator (with addition of series resistor R_i)

For the component values we will be using, the circuit of Figure 5 approximates a differentiator when the input frequency is much less than 19 kHz. When $f \ll 19$ kHz, C_i has an impedance whose magnitude is large compared with R_i. The character of the series R_iC_i circuit at the inverting input of the op amp is, in this case, determined by C_i. (With two impedances in series, the impedance with the larger magnitude dominates.) Hence, with a capacitive input path and a resistive feedback path, the circuit of Figure 5 approximates a differentiator for $f \ll 19$ kHz.

Table 2: Component values for the practical differentiator of Figure 5

Component	Differentiator
C_i	0.0022 μF
R_f	75 kΩ
R_i	3.9 kΩ

For frequencies much larger than 19 kHz, R_i dominates the input path, and the circuit looks like an inverting amplifier, rather than a differentiator. The maximum absolute value of the gain is $R_f\ /R_i$, which is about 20 in this case. Without R_i in the circuit, the absolute value of the differentiator's gain would go well beyond 20 for high frequencies. In summary, the purpose of R_i is to limit the gain at high frequencies (where there is, presumably, only noise content and no signal).

Build the circuit of Figure 5 using the component values of Table 2.

Use the synthesized frequency generator to create a sinewave of amplitude 2 V. Set the frequency f to each of the following values: 1 kHz, 2 kHz, and 4 kHz. For each frequency, observe the input and output signals of the (practical) differentiator on channels 1 and 2 of the oscilloscope. Estimate the phase of the output sinewave relative to that of the input sinewave (that is, the output phase minus the input phase).

Make a table that shows, for each frequency, the input amplitude, the predicted output amplitude, the measured output amplitude, the predicted phase difference, and the observed phase difference. For the predicted amplitude and the predicted phase difference, ignore R_i. (That is to say, the calculations that lead to your predictions should be based on an *ideal* differentiator.)

Use the synthesized frequency generator to create a triangle-wave of amplitude 2 V. Set the frequency first to 1 kHz and then to 2 kHz. Observe the input and output signals on channels 1 and 2 of the oscilloscope.

Lab Report

For the integrator, plot the output amplitude as a function of frequency. The horizontal axis can span the frequencies 1 kHz through 4 kHz. This figure should contain both a solid curve for the theory (based on an ideal integrator) and discrete points for your measured data (using the practical integrator).

Compare your calculation (in the Prelab) of the integrator output to the actual output of your practical integrator for a 1-kHz square-wave.

For the differentiator, plot the output amplitude as a function of frequency. The horizontal axis can span the frequencies 1 kHz through 4 kHz. This figure should contain both a solid curve for the theory (based on an ideal differentiator) and discrete points for your measured data (using the practical differentiator).

Compare your calculation (in the Prelab) of the differentiator output to the actual output of your practical differentiator for a 1-kHz triangle-wave.