California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 12: Transient Response of RC and RL Circuits

Objective

The objective of this experiment is to study the transient response of RC (resistor-capacitor) and RL (resistor-inductor) circuits. The *Single* trigger mode of the oscilloscope is introduced. This is a mode that permits a single trace to be stored and displayed. It is useful for capturing transient signals.

Prelab

First RC Circuit

Consider the circuit of Figure 1. If the switch has been in the closed position for a long time, what will be the voltage v across the capacitor? The capacitor looks like an open circuit to the DC source. With the switch for a long time in the closed position, the current in this circuit will be 0, and the voltage drop across R will be 0. Therefore, the entire 4 V of the source will appear across the capacitor. Of course, this means that there will be a charge on the capacitor.

Let's go back to the moment when the switch was first closed. Let us assign the time $t = 0$ to the exact moment when the switch was closed. We will assume that prior to the switch being closed the voltage v across the capacitor was 0.

The voltage across a capacitor cannot change instantaneously. A capacitor's voltage is proportional to the charge on that capacitor, and charges don't move instantaneously. So how did v change from 0 (at $t = 0$) to 4 V? An equivalent question is: How did the capacitor's charge go from 0 (at $t = 0$) to $4C$ coulombs? (The charge on a capacitor, in coulombs, equals the voltage times the capacitance in farads.) A mathematical description of $v(t)$ during this critical time after the closing of the switch is called the transient response of the circuit. For most circuits of interest in electronics, the duration of a transient response is a small fraction of a second. What follows the transient response is the long-term state (the steady state) that is described above ($v = 4$, for this circuit). Even though the duration of the transient response is short, it is often of importance to characterize this transient response.

We can derive a mathematical expression for the transient response $v(t)$ as outlined here. First, we write a differential equation for $v(t)$ that is valid for $t \geq 0$ with the switch in the closed position. The current through the capacitor equals $C \cdot dv/dt$, and the voltage drop across the resistor therefore equals $RC \cdot dv/dt$. Kirchhoff's Voltage Law becomes:

$$RC\frac{dv}{dt} + v = 4, \quad t \geq 0$$

To this differential equation, we add the initial condition $v(0) = 0$. The differential equation together with this initial condition is called an initial-value problem. The solution to the initial-value problem is the transient response. To find the complete solution, we consider two partial solutions: the *particular* (that is, steady-state) solution v_p and the *homogeneous* solution $v_h(t)$.

The particular solution is valid in the long term, after the transient has died down. In the long term, we expect that nothing will be changing. (The source is DC, so in the long term v should be constant.) Therefore, for the particular solution, $dv_p/dt = 0$, and the differential equation simplifies to $v_p = 4$. (We had already used our intuition about the circuit to find this steady-state solution, but it's nice to know that the mathematics tell us the same thing.)

The homogeneous solution is the solution to the differential equation with the right-hand side set equal to 0:

$$RC\frac{dv_h}{dt} + v_h = 0, \quad t \geq 0$$

For problems of this type, a linear differential equation with constant coefficients, $v_h(t)$ will be of the form:

$$v_h(t) = Ke^{st}$$

which leads to

$$sRC + 1 = 0$$

where we have divided both sides of the equation by Ke^{st} (after taking the derivative). This last equation is called the *characteristic equation*, and this equation tells us that $s = -1/RC$.

The solution to the initial-value problem is the sum of the particular and homogeneous solutions:

$$v(t) = v_p + v_h(t) = 4 + Ke^{-t/(RC)}, \quad t \geq 0$$

where K is an, as yet, undetermined constant. We note that the above equation gives the correct result for the steady-state solution, $v(\infty) = 4$. The homogeneous solution goes to zero for large values of time, leaving only v_p to contribute to the steady-state solution. This outcome was "baked in" when we defined $v_h(t)$ as the solution to the differential equation with the right-hand side set to zero. The role of $v_h(t)$ is to correct v_p during the transient period (the relatively short period that begins at $t = 0$).

The initial condition $v(0) = 0$ will permit us to determine unambiguously what K must be.

$$v(0) = 0 = 4 + Ke^0$$

From this we infer that $K = -4$. Therefore, the solution to the initial-value problem is

$$v(t) = 4\left[1 - e^{-t/(RC)}\right], \quad t \geq 0$$

It is convenient to define a time constant

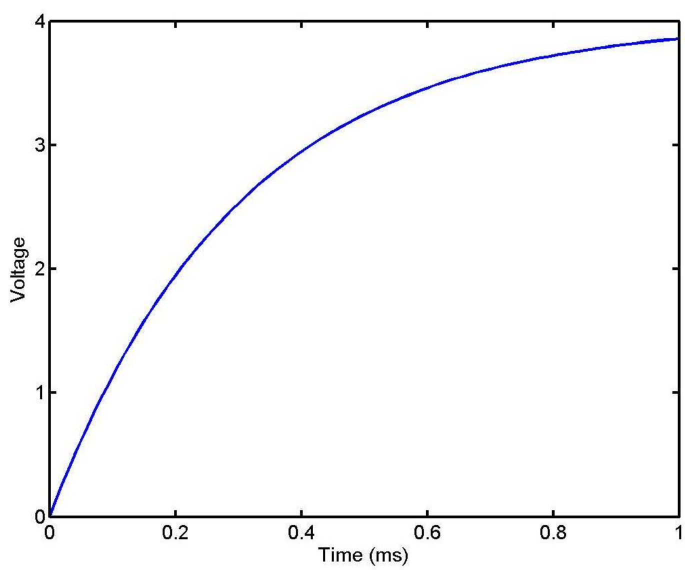
$$\tau = RC$$

Figure 1, closing switch

With R in ohms and C in farads, τ is in seconds. The complete solution may be rewritten as:

$$v(t) = 4\left[1 - e^{-t/\tau}\right], \quad t \geq 0$$

This equation tells us the following. At $t = 0$, v is 0 (as expected). For $t > 0$, $v(t)$ increases monotonically, finally approaching the asymptote $v = 4$. This equation is a mathematical expression of the transient response; it is plotted below:



The time constant τ is convenient for judging roughly the duration of the transient response. At time $t = \tau$ (that is, one time constant after the switch is closed) v reaches the value

$$v(\tau) = 4\left[1 - e^{-\tau/\tau}\right]$$

In other words, τ seconds after the switch is closed, v has attained a value that is a fraction $1 - e^{-1} \cong 0.63$ of its final (steady-state) value. It is common to regard τ as a rough measure of the duration of the transient response.

The time constant for the transient response associated with the closing of the switch in Figure 1 was found above by solving an initial-value problem. There is an intuitive short-cut that reveals the time constant. The thinking for this short-cut goes as follows. The time constant is contained in the homogeneous solution. (The particular solution is a simple constant and says nothing about the shape of the transient response.) We look at Figure 1 and try to imagine how the homogeneous solution falls out of this circuit. The switch is closed (at $t = 0$), and the homogeneous solution is defined as the capacitor voltage with the voltage source set equal to 0. (That is to say, with the right-hand side of the differential equation set to zero.) So we replace, in our minds, the voltage source with a short circuit. We are left with just the capacitor C in parallel with the resistor R. The homogeneous solution at $t = 0$ is:

$$v_h(0) = v(0) - v_p = -4$$

Therefore, $v_h(t)$ is the voltage across the capacitor C as it discharges through R, having started with a value of -4 V at $t = 0$. The time constant associated with this discharge is $\tau = RC$. (Please remember that we replaced the voltage source with a short circuit here for the sake of investigating the homogeneous solution. Of course, for the complete solution of the initial-value problem we cannot ignore the voltage source.)

Exercise: For the circuit of Figure 1 with the parameters of Table 1, what is the time constant associated with the closing of the switch?

Second RC Circuit

Consider the circuit of Figure 2. If the switch has been in the closed position for a long time, what will be the voltage v across the capacitor? The capacitor looks like an open circuit to the DC source. With the switch for a long time in the closed position, the DC source energizes a voltage divider, and the voltage across R_2 (and therefore also across C) is $4R_2/(R_1 + R_2)$.
If the voltage v across the capacitor C in Figure 2 is initially zero, prior to the closing of the switch at $t = 0$, then what is the transient response? In other words, how does v go from 0 at

$t = 0$ to the steady-state value of $4R_2/(R_1 + R_2)$? We could write a differential equation, define an initial-value problem, and solve it by finding and then combining the particular and homogeneous solutions. But we can tell a lot about the transient solution without doing all that work, if we are clever. We already know that the transient response starts at $v(0) = 0$ and leads to a steady-state value of $4R_2/(R_1 + R_2)$. If we can determine the time constant, we know roughly how long this transition takes. So, what is the time constant?

The time constant is defined by the homogeneous solution, which can be envisioned in the circuit by setting the voltage source to zero. With the switch closed and the voltage source set to zero in Figure 2, the resistor R_1 is in parallel with the resistor R_2. The capacitor C in the (homogenous-solution) circuit discharges from its initial value $v_h(0) = -4R_2/(R_1 + R_2)$ through the parallel combination of R_1 and R_2. Therefore, the time constant associated with the closing of the switch in Figure 2 is $\tau = C \cdot R_1R_2/(R_1 + R_2)$.

Exercise: For the circuit of Figure 2 with the parameters of Table 2, what is the time constant associated with the *closing* of the switch?

Now we consider Figure 2 with the switch *opening* at time $t = 0$. Prior to time 0, v will have been $4R_2/(R_1 + R_2)$, assuming that the switch had been in the closed position for a while. The voltage v across that capacitor cannot change instantaneously. Therefore, the initial value is $v(0) = 4R_2/(R_1 + R_2)$. The steady-state solution is $v(\infty) = 0$. What is the time constant associated with the *opening* of the switch? It is emphatically *not* the same as the time constant associated with the *closing* of the switch.

Exercise: For the circuit of Figure 2 with the parameters of Table 2, what is the time constant associated with the *opening* of the switch?

RL Circuit

Consider the circuit of Figure 3. If the switch has been in the closed position for a long time, what will be the current i that passes through L and R_2? The inductor looks like a short circuit to the DC source. (We are neglecting here the resistance that is internal to a physical inductor.) With the switch for a long time in the closed position, (approximately) 4 volts appear across R_2 (and also across R_1), so the current through the inductor will be $4/R_2$.

We now assume that i was 0 and the voltage v across R_2 was 0 prior to the closing of the switch. We'll assign the time $t = 0$ to the exact moment when the switch was closed. What is the transient response? In other words, how does i go from 0 to $4/R_2$ (and v from 0 to 4)? We know that the current through an inductor cannot change instantaneously. So the current i must make a smooth transition to its steady-state value. Since $v = iR_2$, v must make a smooth transition from 0 to 4.

The voltage across the inductor is $L \cdot di/dt$, and the voltage across R_2 is $v = iR_2$. Therefore,

$$L\frac{di}{dt} + R_2 i = 4, \quad t \geq 0$$

This differential equation plus the initial condition $i(0) = 0$ define an initial-value problem. The particular solution is $i_p = 4/R_2$. The homogeneous solution $i_h(t)$ can be found by setting the right-hand side of the above differential equation to 0:

$$L\frac{di_h}{dt} + R_2 i_h = 0, \quad t \geq 0$$

The homogeneous solution is of the form

$$i_h(t) = Ke^{-t/\tau},$$

where the time constant

$$\tau = \frac{L}{R_2}$$

When an inductance in henrys is divided by a resistance in ohms, the result is in seconds. Therefore, τ has the units of seconds, as expected.

The complete solution to the initial-value problem, $i(t) = i_p + i_h(t)$, is

$$i(t) = \frac{4}{R_2}\left[1 - e^{-t/\tau}\right], \quad t \geq 0$$

The voltage across R_2 is

$$v(t) = 4\left[1 - e^{-t/\tau}\right], \quad t \geq 0$$

At $t = 0$, v is 0 (as expected). For $t > 0$, $v(t)$ increases monotonically, finally approaching the asymptote $v = 4$.

Exercise: For the circuit of Figure 3 with the parameters of Table 3, what is the time constant associated with the *closing* of the switch?

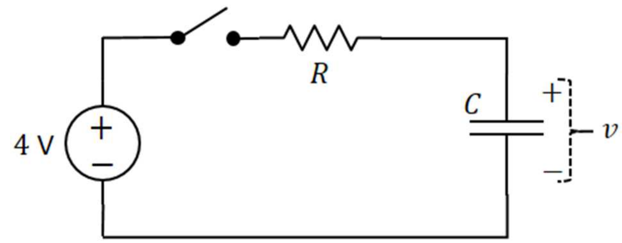


Figure 1: First RC circuit

Now we consider Figure 3 with the switch *opening* at time $t = 0$. The initial value of current is $i(0) = 4/R_2$. The steady-state solution is $i(\infty) = 0$. What is the time constant associated with the opening of the switch? Consider how Figure 3 looks for the homogeneous solution and with the switch open. The voltage source is now a short circuit, but that actually doesn't matter in this particular case since the open switch removes the voltage source from the circuit anyway. Only the inductor L and the resistors R_1 and R_2 remain in the circuit. Beginning at $t = 0$, the inductor's magnetic field and current fade away. (An inductor's current and magnetic field cannot change instantaneously.)

Exercise: For the circuit of Figure 3 with the parameters of Table 3, what is the time constant associated with the *opening* of the switch?

Procedure

First RC Circuit

You will use a decade box for the capacitor C shown in Figure 1. Select a capacitor decade box, and set the correct value according to Table 1. You will use a decade box for the resistor R of Figure 1. Select a resistor decade box, set the correct value according to Table 1, measure and record the actual resistance.

Table 1: Component values for Figure 1

C	R
0.1 μF	3 kΩ

Construct the circuit of Figure 1. For the switch, you will simply have a banana plug that is either connected (closed) or not (open). Set the current limit on the DC source to 0.2 amps.

Connect channel 1 of the oscilloscope to observe the voltage across the resistor/capacitor series combination. (In other words, on channel 1 observe the voltage on the resistor-side of the switch, relative to ground.) Connect channel 2 of the oscilloscope to observe the voltage v across the capacitor. Since the negative side on each channel of the oscilloscope is physically tied to ground (inside the oscilloscope), it is essential that you get the polarity correct when connecting each channel to the circuit.

Prepare the oscilloscope for "single-shot capture". This means that the oscilloscope is to capture a single trace for each channel (as opposed to continuous capture). Appendix A explains how to do to single-shot capture. When configuring the oscilloscope, bear in mind that the capacitor voltage will be zero initially, but will rise to 4 V (and stay there until the 4-V DC source is removed from the circuit). Triggering should be set for a rising edge, and the trigger level should be somewhere between 0 and 4 V.

Make sure that the oscilloscope is set to display both the channel 1 and the channel 2 signals. So you will be capturing a single trace for the channel 1 signal and simultaneously a single trace for the channel 2 signal. (Single trace here means that only one trace per signal is stored and displayed.)

On the trigger menu, select Auto mode. This means that the oscilloscope will be in Auto mode whenever continuous capture is in effect. When you press the Single button, Normal mode will go into effect. After you have obtained a single-shot capture (and, presumably, saved the image of this capture), you will want to push the Run/Stop button in order to put continuous capture back into effect. Doing this will cause the oscilloscope to revert to Auto mode. (On the other hand, if you had selected the Normal mode on the trigger menu, the oscilloscope would always be in Normal mode, both during single-shot capture and during continuous capture.)

As part of the preparation for each new attempt at a single-shot capture of the transient, you should do the following. Open the switch, permit the charge to leak off the capacitor (so that v becomes 0 again), and return to continuous capture (by pressing the Run/Stop button). With this preparation, the oscilloscope should display 0 volts for both channels.

Your switch may experience some bounce (on a very short time scale); this manifests itself as a noisy transient. You will likely have to try closing this switch more than once before you get a smooth transient.

Experiment with different time scales. For your first attempt, you should select a time scale for which one division is roughly the same as the RC time constant.

In the end, you will probably want to use channel 2 as the trigger source. However, you should experiment also with channel 1 as the trigger source. You should also experiment with different trigger levels. (But for this circuit, a successful single-shot acquisition requires that the trigger

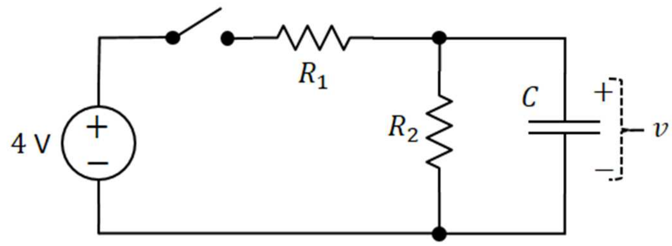


Figure 2: Second RC circuit

level be between 0 and 4 V.) With this experimentation, you should gain a better understanding of the triggering mechanism.

Make an experimental determination of the RC time constant. This is the time duration between the start of the transient (when the switch is closed) to the time when v is a fraction $(1 - e^{-1}) \cong 0.63$ of the way from its starting value to its ending value. In this case, the starting and ending values are 0 V and 4 V, respectively.

You should use cursors to make an accurate determination of the time constant. (You'll be using the cursor menu on the oscilloscope.) You should first calculate the voltage that is 0.63 of the way from $v(0)$ to $v(\infty)$. For example, in the present case, this is $0 + 0.63(4 - 0) = 2.52$ V. Place an amplitude (horizontal) cursor at this level. Move one time (vertical) cursor to the time at which the transient starts and a second time cursor to the time at which the transient response intersects the amplitude cursor. The time between these two time cursors is the experimentally determined time constant.

Second RC Circuit

You will use a decade box for the capacitor C shown in Figure 2. Select a capacitor decade box, and set the correct value according to Table 2. You will use a decade box for each of resistors R_1 and R_2 in Figure 2. Select resistor decade boxes, set the correct values according to Table 2, measure and record the actual resistances.

Table 2: Component values for Figure 2

C	R_1	R_2
0.1 µF	1 kΩ	3 kΩ

Construct the circuit of Figure 2. A banana plug will serve as the switch. Set the current limit on the DC source to 0.2 amps.

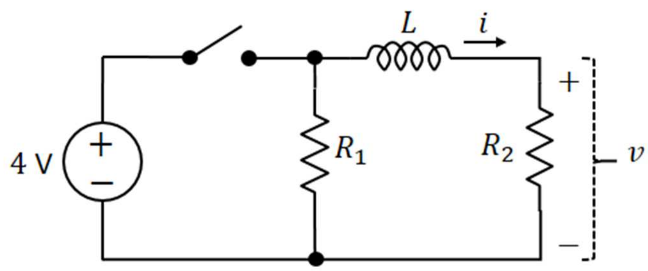


Figure 3: *RL* circuit

Connect channel 1 of the oscilloscope to observe the voltage v across the capacitor. You won't use channel 2 while experimenting with the circuit of Figure 2.

Prepare the oscilloscope for single-shot capture of the transient caused by the *closing* of the switch. In this case, v will rise from 0 V to the final voltage across R_2. (After the transient settles, the capacitor looks like an open circuit to the DC source, and the final voltage across C and R_2 is given by a simple voltage division.) Triggering should be set for a rising edge. The trigger level should be set to a value intermediate between 0 V and the final voltage.

Measure the time constant associated with the *closing* of the switch. Use cursors.

Now prepare the oscilloscope for single-shot capture of the transient caused by the *opening* of the switch. In this case, v will fall to 0 V. Triggering should be set for a falling edge. Make sure that the trigger level is set to an appropriate value (a value between the initial and final voltages).

Measure the time constant associated with the *opening* of the switch. Please note that this time constant is different from the previously measured time constant associated with the *closing* of the switch.

RL Circuit

You will use a decade box for the inductor L shown in Figure 3. Set the correct value according to Table 3. You will use a decade box for each of resistors R_1 and R_2 in Figure 3. Select resistor decade boxes, set the correct values according to Table 3, measure and record the actual resistances.

Table 3: Component values for Figure 3

L	R_1	R_2
300 mH	3 kΩ	3 kΩ

Construct the circuit of Figure 3. A banana plug will serve as the switch. Set the current limit on the DC source to 0.2 amps.

Connect channel 1 of the oscilloscope to observe the voltage v across R_2. This voltage is proportional to the current i through the inductor. You won't use channel 2 while experimenting with the circuit of Figure 3.

Prepare the oscilloscope for single-shot capture of the transient caused by the *closing* of the switch. In this case, v will rise from 0 V to the final voltage across R_2. (This final voltage will be close to 4 V, but it will be a little less than 4 V because there is a small voltage drop across the resistance that is internal to the inductor.) Triggering should be set for a rising edge. The trigger level should be set to an appropriate value.

Measure the time constant associated with the *closing* of the switch.

Now prepare the oscilloscope for single-shot capture of the transient caused by the *opening* of the switch. In this case, v will fall to 0 V. Triggering should be set for a falling edge.

Measure the time constant associated with the *opening* of the switch. Please note that this time constant is different from the previously measured time constant associated with the *closing* of the switch.

Lab Report

Are your experimentally determined time constants approximately equal to their theoretical values?

Answer the following two questions about capacitors. Can the voltage across a capacitor change abruptly (instantaneously)? Can the current through a capacitor change abruptly?

Answer the following two questions about inductors. Can the voltage across an inductor change abruptly? Can the current through an inductor change abruptly?

Appendix A: Oscilloscope Single-Shot Capture of a Transient Voltage

Triggering should be set for an appropriate level (somewhere between the voltage at the start of the transient and the expected asymptotic value). The trigger slope should be set for the expected slope (rising or falling).

The horizontal scale should be set to an appropriate value.

With the oscilloscope still in continuous capture mode, the displayed voltage should equal the expected initial value of voltage. The Run/Stop button should be green still, indicating continuous capture mode.

Push the Single button. It should now turn green. This indicates that the oscilloscope is waiting for the right trigger conditions.

Once the trigger conditions are met (by the closing of a switch, for example), the transient should be captured. Since you are no longer in continuous capture mode, this transient should remain displayed until you instruct the oscilloscope otherwise.