# California State University, Fresno
# Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 13: Driven *RC* and *RLC* Circuits

## Objective

The objective of this experiment is to see how $RC$ and $RLC$ (resistor-inductor-capacitor) circuits respond to square-wave excitation.

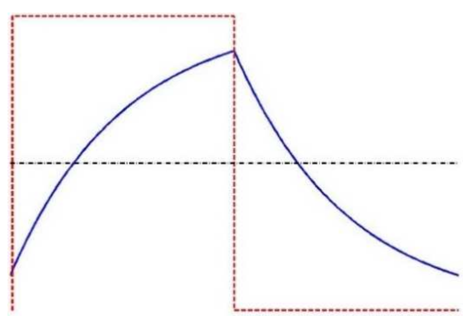
## Prelab

*RC Circuit*

Consider the circuit of Figure 1. The voltage $v(t)$ across the capacitor and the voltage $x(t)$ supplied by the synthesized frequency generator are related through the following differential equation:

$$RC\frac{dv}{dt} + v = x$$

In this experiment $x(t)$ will be, in every case, a square-wave. In response to the square-wave excitation $x(t)$, the capacitor's voltage $v(t)$ will look like this:



The square-wave (dotted line) has the two values $+5$ V and $-5$ V in this experiment. The capacitor's voltage (solid curve) responds to a sudden change in $x$. When $x$ changes suddenly from $-5$ V to $+5$ V, the direction of the current reverses and $dv/dt$ changes sign. (However, $v$ does not change instantaneously.) As long as $x = +5$, $v$ moves toward $+5$ at a pace defined by the time constant $\tau = RC$. When $x$ changes suddenly from $+5$ to $-5$, $dv/dt$ changes sign again. Thus, before arriving at $+5$, $v$ begins to move toward $-5$.

The time constant can be determined from the homogeneous equation (obtained by setting the right-hand side of the original differential equation to 0):

$$RC\frac{dv_h}{dt} + v_h = 0$$

The $v_h(t)$ that solves this homogeneous equation is of the form:

$$v_h(t) = Ke^{st}$$

which leads to

$$sRC + 1 = 0$$

where we have divided both sides of the equation by $Ke^{st}$ (after taking the derivative). This last equation is called the *characteristic equation*, and this equation tells us that $s = -1/RC$ and that $v_h(t)$ can therefore be written in the form

$$v_h(t) = Ke^{-t/\tau}$$

where $\tau = RC$. This $v_h(t)$ is not the complete solution for $v$ because it does not contain the particular solution (which accounts for the actual value of $x$). However, the homogeneous solution answers the question about the time constant.

**Exercise:** For the *RC* circuit of Figure 1 with the component values of Table 1, calculate the time constant.

*RLC Circuit*

Consider the circuit of Figure 2. This circuit contains two reactive elements (capacitors and inductors), and therefore the circuit is governed by a second-order differential equation. The excitation $x(t)$ is a square-wave with an amplitude of 5 V. We are interested in the voltage drop $v(t)$ across the resistor (and also across the capacitor).

We find the differential equation for this circuit with the following reasoning. With $x(t)$ denoting the voltage supplied by the signal generator and $i_L(t)$ denoting the current through the inductor (in the direction from left to right), then

$$x - v = L\frac{di_L}{dt}$$

Kirchhoff's Current Law for the node where $R$, $L$ and $C$ meet can be written as:

$$i_L = C\,\frac{dv}{dt} + \frac{v}{R}$$

We can take the derivative of both sides of this equation for $i_L$ and then multiple both sides by $L$. The result is

$$L\,\frac{di_L}{dt} = LC\,\frac{d^2v}{dt^2} + \frac{L}{R}\,\frac{dv}{dt}$$

The left-hand side of this equation may be replaced by $x - v$, and then we have

$$x - v = LC\,\frac{d^2v}{dt^2} + \frac{L}{R}\,\frac{dv}{dt}$$

Rearranging terms, this is

$$LC\,\frac{d^2v}{dt^2} + \frac{L}{R}\,\frac{dv}{dt} + v = x$$

This is the second-order, linear, constant-coefficient differential equation that governs the voltage $v$. As usual, we can solve a differential equation of this type by calculating a homogeneous solution $v_h$ and a particular solution and finally stitching these two partial solutions together. However, for this experiment we only need examine the homogeneous solution to understand the character of the transient response.

$$LC\,\frac{d^2v_h}{dt^2} + \frac{L}{R}\,\frac{dv_h}{dt} + v_h = 0$$

With the substitution

$$v_h = Ke^{st}$$

and then dividing through by $Ke^{st}$, we get the characteristic equation

$$LCs^2 + \frac{L}{R}s + 1 = 0$$

This is a quadratic equation and will have two roots. The discriminant

$$\left(\frac{L}{R}\right)^2 - 4LC$$

determines the character of the two roots and the character of the transient response. Taking into consideration that *R*, *L* and *C* are positive, three possibilities exist:

| when discriminant is: | 2 roots are: | character of transient response: |
|---|---|---|
| 0 | identical, real and negative | critically damped |
| positive | unequal, real and negative | overdamped |
| negative | complex conjugate pair | underdamped |

In the third case, the real parts of the two complex roots are negative.

An underdamped $v$ will contain a decaying sinusoid. You can see this from the following consideration. In the underdamped case, the roots of the characteristic equation are $s_1 = \alpha + j\beta$ and $s_2 = \alpha - j\beta$, where $\alpha < 0$.

$$Ke^{s_1 t} + Ke^{s_2 t} = Ke^{\alpha t}\left[e^{j\beta t} + e^{-j\beta t}\right] = 2Ke^{\alpha t}\cos(\beta t)$$

$\alpha$ is negative, so this is a *decaying* sinusoid.

**Exercise:** Consider the *RLC* circuit of Figure 2. For the *first* parameter set of Table 2, determine the character of the transient response. For the *second* parameter set of Table 2, determine the character of the transient response. For $C = 0.01$ μF and $L = 100$ mH, determine the resistance $R_c$ corresponding to a critically damped transient response.

**Procedure**

*RC Circuit*

You will use a decade box for the capacitor $C$ shown in Figure 1. Select a capacitor decade box, and set the correct value according to Table 1. You will use a decade box for the resistor $R$ of Figure 1. Select a resistor decade box, set the correct value according to Table 1, measure and record the actual resistance.

**Table 1:** Component values for Figure 1

| ***C*** | ***R*** |
|---|---|
| 0.025 μF | 2 kΩ |

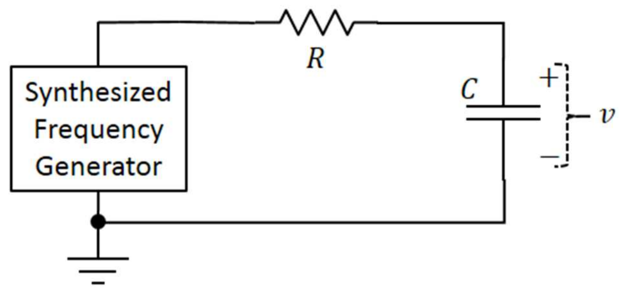
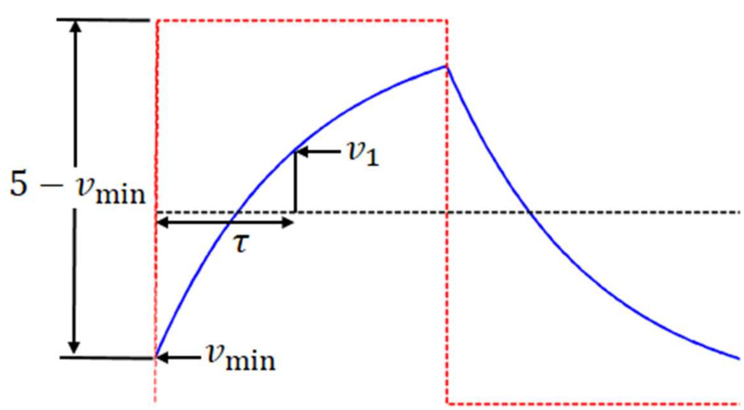



**Figure 1:** $RC$ circuit

Construct the circuit of Figure 1. Set the synthesized frequency generator to produce a square-wave with amplitude 5 V and frequency 5 kHz.

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage $v$ across the capacitor.

Make an experimental determination of the $RC$ time constant. You can do this with the following procedure. First, measure the minimum value $v_{\mathrm{min}}$ of $v$. (This is the negative value of $v$ at the moment when the square-wave excitation changes from $-5$ V to $+5$ V.) Then calculate $v_1 = v_{\mathrm{min}} + 0.63(5 - v_{\mathrm{min}})$. (Remember that $v_{\mathrm{min}}$ is negative, so $5 - v_{\mathrm{min}}$ means adding a positive value to 5.) On the oscilloscope screen, find that first place after $v$ starts its climb where $v$ reaches $v_1$. This is where $v$ reaches a fraction $1 - e^{-1} \cong 0.63$ of the way from $v_{\mathrm{min}}$ to $+5$. (Of course, $v$ never makes it all the way to $+5$ before the square-wave reverses sign, causing $v$ to stop dead in its tracks and then start to head in the opposite direction.) The time taken by $v$ in going from $v_{\mathrm{min}}$ to $v_1$ is the time constant. Cursors will help you make this measurement. This experimental determination of the time constant is illustrated below.




$$v_1 = v_{\mathrm{min}} + 0.63(5 - v_{\mathrm{min}})$$

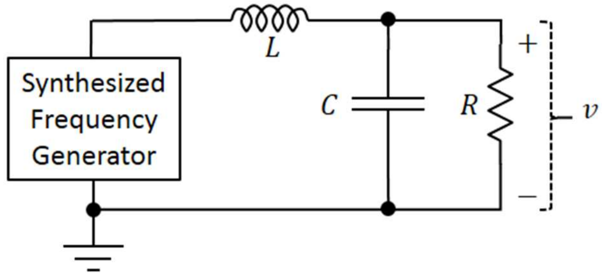



**Figure 2:** $RLC$ circuit

*RLC Circuit*

You will use a decade box for each of the following components of Figure 2: the inductor $L$, the capacitor $C$, and the resistor $R$. You will implement this circuit three times, once for each parameter set given in Table 2. (The resistance $R_c$ in the third parameter set should be available to you from your Prelab solutions.) For $R$, measure and record the actual resistance for each parameter set.

**Table 2:** Component values for Figure 2

| Parameter Set | $L$ | $C$ | $R$ |
|---|---|---|---|
| 1 | 100 mH | 0.01 μF | 1 kΩ |
| 2 | 100 mH | 0.01 μF | 4 kΩ |
| 3 | 100 mH | 0.01 μF | $R_c$ |

Construct the circuit of Figure 2. Set the synthesized frequency generator to produce a square-wave with amplitude 5 V and frequency 1 kHz.

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage $v$ across the resistor. The oscilloscope display should show enough detail that you can identify the character of the transient response.

Repeat the above procedure using the component values for the Parameter Set 2 row of Table 2.

Repeat the above procedure using the component values for the Parameter Set 3 row of Table 2. In this case, $R = R_c$, so we expect critical damping.

For second-order linear circuits, we generally prefer critical damping. When a transient response is overdamped, the voltage is slow to attain its final value. With underdamping, on the other hand, the voltage moves quickly toward its final value, but there is overshoot (also called "ringing"), which is often undesirable. A critically damped response moves toward the final voltage more quickly than does an overdamped response, but critical damping also avoids overshoot.

## Lab Report

For the *RC* circuit, do your calculated and experimentally determined time constants agree?

For each parameter set of the *RLC* circuit, does your expectation about the nature of the transient response match your observations?