

California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 14: Kirchhoff's Circuit Laws for Sinusoidal Excitation

Objective

The first objective is to test Kirchhoff's Voltage Law (KVL) for a series *RLC* circuit that is excited by a sinusoidal voltage. The second objective is to test Kirchhoff's Current Law (KCL) for a parallel *RLC* circuit with sinusoidal excitation.

Prelab

Sinusoids

A sinusoid $x(t)$ may be modeled as

$$x(t) = A_x \cos(2\pi ft + \theta_x)$$

The important parameters are: frequency f , amplitude A_x , and phase θ_x . (Sometimes people use the word "phase" to mean the entire argument of the cosine function: $2\pi ft + \theta_x$. That entire argument is a function of time. In this experiment, we regard the phase as just the constant θ_x .) The period T is the reciprocal of the frequency; that is, $T = 1/f$.

Phase is a relative quantity; its value depends on how the zero of the time axis is assigned. For example, we can regard the variable t as Pacific Standard Time (PST), measured in seconds. We could also define a time variable t' as Pacific Daylight Time (PDT), measured in seconds. If we change from the one time variable to the other, then the zero of time shifts by 3600 seconds.

$$t' = t + \tau$$

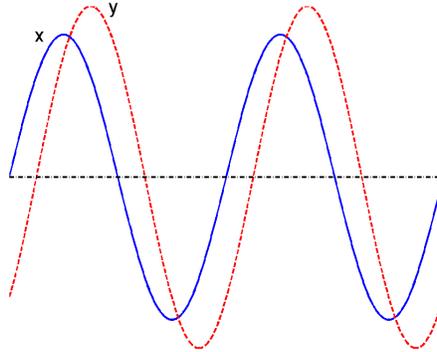
where $\tau = 3600$ when the change takes place in the spring. Now we model the above sinusoid like this:

$$x(t') = A_x \cos(2\pi ft' + \theta_x')$$

where the new phase $\theta_x' = \theta_x - 2\pi f\tau$. The frequency, period, and amplitude are unaffected by the change in time variable.

Phase Difference Between Two Sinusoids of a Common Frequency

When two sinusoids having the same frequency appear on the oscilloscope display, we must be able to identify the phase difference between the sinusoids.



The first sinusoid (labelled “x”) is the $x(t)$ discussed above. The second (labelled “y”) has the mathematical form

$$y(t) = A_y \cos(2\pi f t + \theta_y)$$

with frequency f , amplitude A_y , and phase θ_y (according to PST). If we use PDT,

$$y(t') = A_y \cos(2\pi f t' + \theta_y')$$

where $\theta_y' = \theta_y - 2\pi f \tau$. Both of these sinusoids have the same frequency. If they didn't have the same frequency, we couldn't define a constant phase difference between them.

The phase difference between $y(t)$ and $x(t)$ is $\theta_y - \theta_x$. This phase difference is the same, independent of where we place the zero of the time axis.

$$\theta_y' - \theta_x' = (\theta_y - 2\pi f \tau) - (\theta_x - 2\pi f \tau) = \theta_y - \theta_x$$

We also call $\theta_y - \theta_x$ the phase of y relative to x . When we say the phase of x relative to y , we mean $\theta_x - \theta_y$.

To summarize, phase *difference* (unlike phase itself) has a physical significance that is independent of how we define the time axis. In the laboratory, we shall generally be interested only in phase differences, not the individual phase terms.

Table 1 lists a procedure for measuring the phase difference between two sinusoids that are displayed on an oscilloscope screen.

Table 1: Procedure for measuring phase difference of two sinusoids

1. For best accuracy, use AC coupling for both channels of the oscilloscope.
2. Adjust the 0-V reference line to be the oscilloscope's horizontal axis for both channels.
3. Select one positive-going zero-crossing for each sinusoid. These two positive-going zero-crossings should be separated in time by no more than one-half the period.
4. Measure the time difference Δt between these two positive-going zero-crossings.
5. From Δt and the period T , calculate the absolute value of the phase difference in radians:

$$|\theta_y - \theta_x| = 2\pi \frac{\Delta t}{T}$$

6. Attach the correct algebraic sign to the phase difference. If the sinusoid $y(t)$ *leads* the sinusoid $x(t)$, then $\theta_y - \theta_x$ is positive. If $y(t)$ *lags*, then $\theta_y - \theta_x$ is negative.

We should be clear about what it means for one sinusoid to lead or lag another. If the positive-going zero-crossing of $y(t)$ is to the right of the positive-going zero-crossing of $x(t)$, as shown in the example plot above, then $y(t)$ *lags* $x(t)$ and $\theta_y - \theta_x$ is negative. On the other hand, if $y(t)$ is to the left of $x(t)$ then $y(t)$ *leads* $x(t)$ and $\theta_y - \theta_x$ is positive. For these definitions to be useful, it is essential that $\Delta t \leq T/2$. (If this inequality does not hold, then you need to reselect at least one positive-going zero-crossing.)

Kirchhoff's Circuit Laws

Kirchhoff's circuit laws also apply to a circuit containing reactive components (capacitors and inductors) that is excited by a sinusoid. In this case, each voltage and current is represented by a phasor. For the sinusoid $x(t)$, its phasor \mathbf{X} can be written as a complex number in polar form as:

$$\mathbf{X} = A_x e^{j\theta_x}$$

where A_x and θ_x are the amplitude and phase of $x(t)$. In this mathematical form, θ_x will be in units of radians. A_x will have the same units as $x(t)$. The phasor can also be written as:

$$\mathbf{X} = A_x \angle \theta_x$$

In this mathematical form, θ_x will usually be in units of degrees.

For a circuit with reactive components that is excited by a sinusoid, Kirchhoff's circuit laws are:

KVL: The sum of all the voltage phasors around any closed path in a circuit equals 0.

KCL: The sum of all the current phasors at any node in a circuit equals 0.

Of course, these sums are computed using complex numbers. Although we typically write a phasor as a complex number in polar form, for the sake of calculating a sum of phasors, it will generally be convenient to use the rectangular form for the phasors. Better yet, we can ask MATLAB to sum complex numbers for us.

In addition to Kirchhoff's circuit laws, we need the V - I relationship (between the voltage phasor V and the current phasor I) for each component in a circuit.

<u>Component</u>	<u>V-I Relationship</u>
Resistor	$V = RI$
Inductor	$V = j\omega LI$
Capacitor	$V = \frac{I}{j\omega C}$

When doing a circuit analysis with sinusoids, we usually define the phase of the source as 0. For each voltage and current, the calculated phase then really represents the phase of that voltage or current relative to the phase of the source. For example, if a component voltage has a calculated phase of 40° , then this means that this component voltage has an actual phase that is larger than the phase of the source by 40° . (On the oscilloscope, the component voltage leads the source.) If a component current has a calculated phase of -40° , then this means that this component current has an actual phase that is smaller than the phase of the source by 40° . (On the oscilloscope, the component current lags the source.) It does no harm to define the phase of the source as 0, since only phase *differences* have physical significance.

Exercise: Consider the series RLC circuit of Figure 1 with the source a 1-kHz sinusoid having an amplitude of 5 V. Regard the source voltage as having zero phase. Calculate the amplitude and phase (in degrees) of the following quantities: the voltage across the resistor, the voltage across the inductor, and the voltage across the capacitor.

Exercise: Consider the parallel RLC circuit of Figure 2 with the source a 1-kHz sinusoid having an amplitude of 5 V. Regard the source voltage as having zero phase.

Calculate the amplitude and phase (in degrees) of the following quantities: the current through the resistor, the current through the inductor, and the current through the capacitor. Calculate also the amplitude and phase of the current supplied by the source.

Procedure

Series RLC Circuit

You will use a decade box for the capacitor C shown in Figure 1. Select a capacitor decade box, and set the correct value according to Table 1. You will use a decade box for the resistor R of Figure 1. Select a resistor decade box, set the correct value according to Table 1, measure and record the actual resistance. Set the inductor decade box in accord with Table 1.

Table 1: Component values for the circuits of Figures 1 and 2

Component	Value
R	5 k Ω
L	900 mH
C	0.08 μ F

Construct the circuit of Figure 1. Set the synthesized frequency generator to produce a sinewave with amplitude 5 V and frequency 1 kHz.

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage across the capacitor. Since the negative side on each channel of the oscilloscope is physically tied to ground (inside the oscilloscope), it is essential that you get the polarity correct when connecting each channel to the circuit. Use AC coupling for both channels. From this display, you should estimate the phase of the capacitor voltage relative to the source voltage. Make sure you assign the correct sign to this phase difference.

You could also measure the amplitude of the capacitor voltage on the oscilloscope. But you will get a more accurate measurement by using the AC voltmeter. Remember that the AC voltmeter displays rms voltage. You must convert this rms voltage to an amplitude.

You will want to measure the amplitude and phase (relative to the source voltage) of the voltage across the resistor and the amplitude and phase of the voltage across the inductor. There is, however, a problem, as you may surmise. The negative lead for the oscilloscope channel is

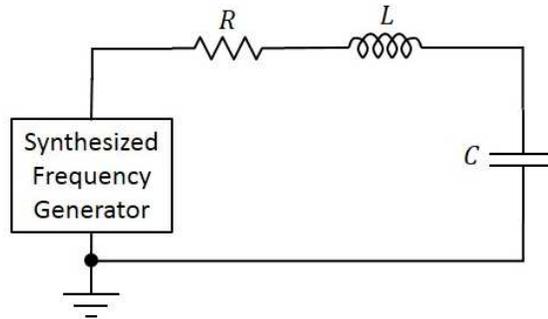


Figure 1: Series *RLC* circuit

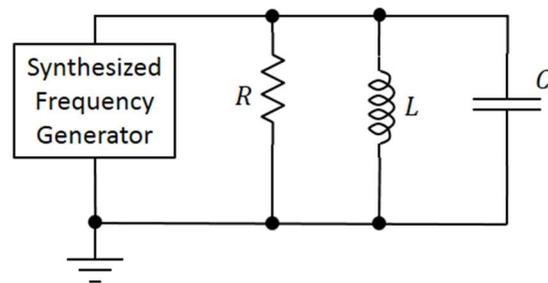


Figure 2: Parallel *RLC* circuit

connected to Earth ground (inside the instrument). To solve this problem, you can reorder the components in this series circuit before making each set of measurements. In every case, channel 2 of the oscilloscope should be observing a voltage between two points, one of which is grounded.

Use the oscilloscope to determine the phase of the voltage across the inductor (relative to the source voltage). Use the AC voltmeter to measure the rms voltage across the inductor. (And then convert that rms voltage to an amplitude.) Repeat that procedure for the voltage across the resistor (after reordering the circuit).

KVL should be satisfied, at least approximately, by your measurements. The sum of the three component phasors (one each for the resistor, inductor and capacitor) should approximate the source phasor, which equals, in this case, approximately $5\angle 0^\circ$ volts.

Parallel RLC Circuit

Use the components of Table 1 to construct the parallel *RLC* circuit of Figure 2. Set the synthesized frequency generator to produce a sinewave with amplitude 5 V and frequency 1 kHz.

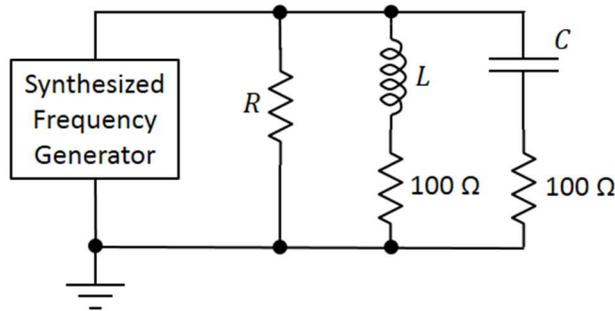


Figure 3: Parallel RLC circuit with test resistors

Use the AC ammeter to measure the rms current through each component (resistor, inductor, and capacitor) of the parallel RLC circuit. Also measure the rms current supplied by the synthesized frequency generator. Convert all of the rms currents to amplitudes.

For a complete picture, we now need to determine the phase of each current relative to the source voltage. Only the oscilloscope can provide these relative phases. But an oscilloscope displays voltages, not currents. Fortunately, we know how to create a voltage that is proportional to any given current.

Place a $100\text{-}\Omega$ test resistor in series with the inductor, such that one end of the test resistor is connected to ground. Also place a $100\text{-}\Omega$ test resistor in series with the capacitor, as shown in Figure 3. These test resistors were selected with a small enough value ($100\ \Omega$) that they will not significantly change the currents in the parallel RLC circuit. These test resistors are also large enough that an adequate amplitude appears across them, for the sake of measuring phase difference. (For a given current, the voltage across a resistor is proportional to the resistance.)

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage across the test resistor that is in series with the capacitor. Use AC coupling for both channels. From this display, you should estimate the phase of the capacitor voltage relative to the source voltage. Make sure you assign the correct sign to this phase difference. For a resistor, voltage and current are in phase with each other. Therefore, the measured phase of the (capacitor-branch) *test-resistor voltage* relative to the source voltage is also the phase of the *capacitor current* relative to the source voltage.

Repeat the above procedure to determine the phase of the inductor current relative to the source voltage.

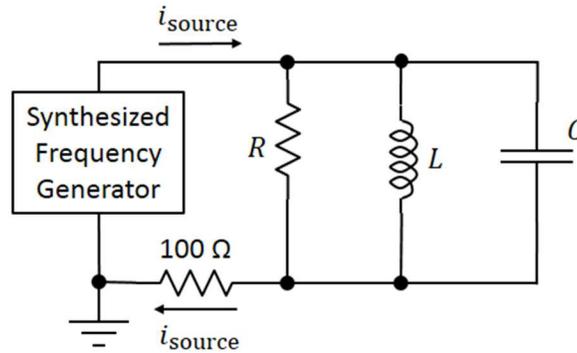


Figure 4: Configuration for measuring source current phase relative to source voltage

We now want to determine the phase of the current supplied by the source relative to the source voltage. This can be done with the circuit configuration shown in Figure 4. A new $100\text{-}\Omega$ test resistor is added in series with the return path from the RLC parallel combination to the source. (The other two test resistors can be removed, as they are no longer needed.)

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage across the test resistor shown in Figure 4. Use AC coupling for both channels. From this display, you should estimate the phase of the test-resistor voltage relative to the source voltage. Make sure you assign the correct sign to this phase difference. For a resistor, voltage and current are in phase with each other. Therefore, the measured phase of the *test-resistor voltage* relative to the source voltage is also the phase of the *source current* relative to the source voltage.

KCL should be satisfied, at least approximately, by your measurements. The sum of the three component phasors (one each for the resistor current, the inductor current and the capacitor current) should approximate the source-current phasor.

Lab Report

Did you find that KVL is satisfied, at least approximately, for the series RLC circuit?

Did you find that KCL is satisfied, at least approximately, for the parallel RLC circuit?