California State University, Fresno
Department of Electrical and Computer Engineering

ECE 90L Principles of Electrical Circuits Laboratory
Experiment No. 15: Additional Circuits with Sinusoidal Excitation

Objective

The objective of this experiment is to test circuits with reactive components and a sinusoidal source.

Prelab

A number of techniques have been found useful for analyzing resistive circuits (containing only resistors and sources). These techniques include: voltage division, current division, two-terminal network, Thévenin equivalent, and superposition.

All of those same techniques can be used, with some modification, to analyze circuits that contain reactive components (capacitors and inductors) as well as resistors and sources. When the source is sinusoidal, the required modification is that phasors be used to represent voltages and currents. Therefore, the calculations will employ complex numbers, rather than just real numbers.

In this experiment, we look at three examples: voltage division in an *RC* circuit, voltage division in an *RL* circuit, and a two-terminal network that contains an inductor, a capacitor and a resistor.

Voltage Division: RC Circuit

Exercise: Consider the *RC* circuit of Figure 1 with parameters of Table 1. A 1-kHz sinewave with amplitude 4 V is provided by the synthesized frequency generator. Model this source voltage as having a phase of zero. Treating this circuit as a voltage divider, calculate the amplitude and phase of the voltage across the capacitor and the amplitude and phase of the voltage across the resistor.

Voltage Division: RL Circuit

Exercise: Consider the *RL* circuit of Figure 2 with parameters of Table 2. A 1-kHz sinewave with amplitude 4 V is provided by the synthesized frequency generator. Model this source voltage as having a phase of zero. Treating this circuit as a voltage divider, calculate the amplitude and phase of the voltage across the inductor and the amplitude and phase of the voltage across the resistor.

Two-Terminal Network

A two-terminal network containing resistors, capacitors and inductors will have an impedance $Z(\omega)$ that is a function of the angular frequency ω. (Angular frequency ω, in rad/s, is related to cyclical frequency f, in Hz, by $\omega = 2\pi f$.)

For any two-terminal *RLC* network, no matter how complicated, a simpler network may be found that is equivalent to the original network at any given angular frequency ω_0. (ω_0 is the angular frequency of the source that will eventually be connected to the two-terminal network.) We can write $Z(\omega_0)$ like this:

$$Z(\omega_0) = R(\omega_0) + jX(\omega_0)$$

where $R(\omega_0)$ is the real part of $Z(\omega_0)$ and $X(\omega_0)$, called the reactance, is the imaginary part of $Z(\omega_0)$. Both $R(\omega_0)$ and $X(\omega_0)$ depend on ω_0. (For any two-terminal network containing at least one reactive component, the impedance changes when the frequency changes.)

If $X(\omega_0) < 0$, then the equivalent network is a series combination of a resistor $R_{\text{eq}} = R(\omega_0)$ and a capacitor

$$C_{\text{eq}} = \frac{-1}{\omega_0\, X(\omega_0)}, \qquad X(\omega_0) < 0$$

If $X(\omega_0) > 0$, on the other hand, then the equivalent network is a series combination of the resistor $R_{\text{eq}} = R(\omega_0)$ and an inductor

$$L_{\text{eq}} = \frac{X(\omega_0)}{\omega_0}, \qquad X(\omega_0) > 0$$

The simpler network is equivalent to the original two-terminal network in the sense that the simpler network can substitute for the original network in any circuit whose only source is a sinusoidal source of angular frequency ω_0. The voltage v and current i at the terminals (as shown in Figure 3) will be unchanged by the substitution.

It is important to notice that the equivalent network (of a two-terminal network) will change if the frequency changes. Also, if the circuit (to which the original two-terminal network is attached) has a non-sinusoidal source, such as a square-wave or triangle-wave source, then the original network cannot be replaced with a simpler network.

Exercise: Consider the two-terminal network of Figure 3 with parameters of Table 3. For the case where this network is excited by a 2-kHz sinusoid, find the equivalent network (the series combination of R_{eq} and either C_{eq} or L_{eq}).

Exercise: In Figure 4 the two-terminal network of Figure 3 with the parameters of Table 3 is placed in a circuit. The source is a 2-kHz sinusoid with a 4-V amplitude. Model this source voltage as having a phase of zero. $R_S = 2$ kΩ. Find the amplitude and phase of v and i.

Procedure

Voltage Division: RC Circuit

You will use a decade box for the capacitor C shown in Figure 1. Select a capacitor decade box, and set the correct value according to Table 1. You will use a decade box for the resistor R of Figure 1. Select a resistor decade box, set the correct value according to Table 1, measure and record the actual resistance.

Table 1: Component values for the circuit of Figure 1

Component	Value
R	4 kΩ
C	0.03 μF

Construct the circuit of Figure 1. Set the synthesized frequency generator to produce a sinewave with amplitude 4 V and frequency 1 kHz.

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage across the capacitor. Since the negative side on each channel of the oscilloscope is physically tied to ground (inside the oscilloscope), it is essential that you get the polarity correct when connecting each channel to the circuit. From the oscilloscope display, estimate the phase of the capacitor voltage relative to the source voltage. Make sure you assign the correct sign to this phase difference.
Measure the rms voltage across the capacitor using an AC voltmeter. Convert this to an amplitude. (You could determine the amplitude of the capacitor voltage using the oscilloscope, but the AC voltmeter provides better accuracy.)

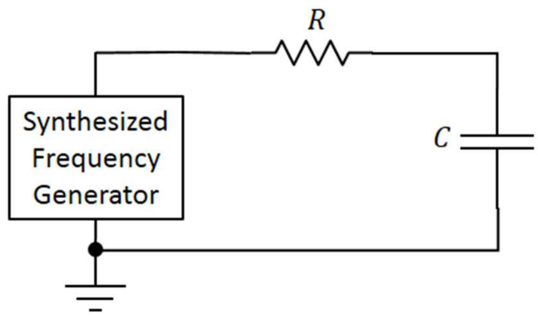

Figure 1: RC circuit

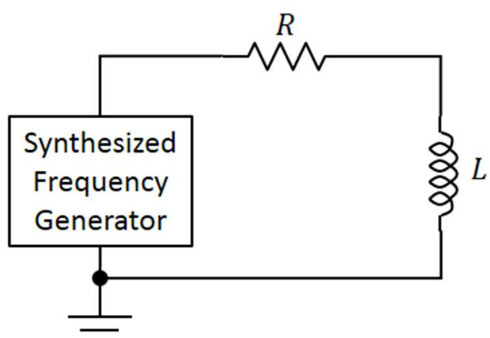

Figure 2: RL circuit

Have the capacitor and resistor swap places in this circuit, so that the resistor now has one end grounded. Repeat the above procedure to determine the amplitude of the resistor voltage and the phase of this voltage relative to the source voltage.

Voltage Division: RL Circuit

You will use a decade box for the inductor L shown in Figure 2. Set the correct value according to Table 2. You will use a decade box for the resistor R of Figure 2. Select a resistor decade box, set the correct value according to Table 2, measure and record the actual resistance.

Table 2: Component values for the circuit of Figure 2

Component	**Value**
R	4 kΩ
L	300 mH

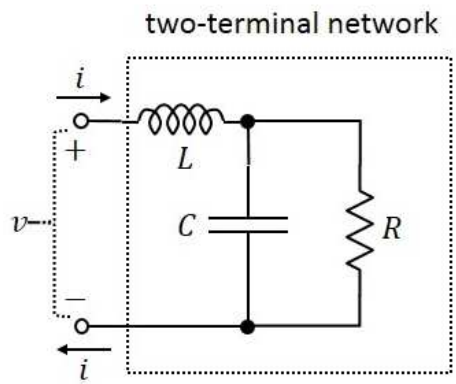

Figure 3: Two-terminal network

Construct the circuit of Figure 2. Set the synthesized frequency generator to produce a sinewave with amplitude 4 V and frequency 1 kHz.

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage across the inductor. From the oscilloscope display, estimate the phase of the inductor voltage relative to the source voltage. Make sure you assign the correct sign to this phase difference. Measure the rms voltage across the inductor using an AC voltmeter. Convert this to an amplitude.

Have the inductor and resistor swap places in this circuit, so that the resistor now has one end grounded. Repeat the above procedure to determine the amplitude of the resistor voltage and the phase of this voltage relative to the source voltage. Make sure you assign the correct sign to this phase difference.

Two-Terminal Network

You will use a decade box for each of the following components of Figure 3: the inductor L, the capacitor C, and the resistor R. Set the values given in Table 3. For R, measure and record the actual resistance.

Table 3: Component values for the two-terminal network of Figure 3

Component	Value
L	100 mH
C	0.022 μF
R	5 kΩ

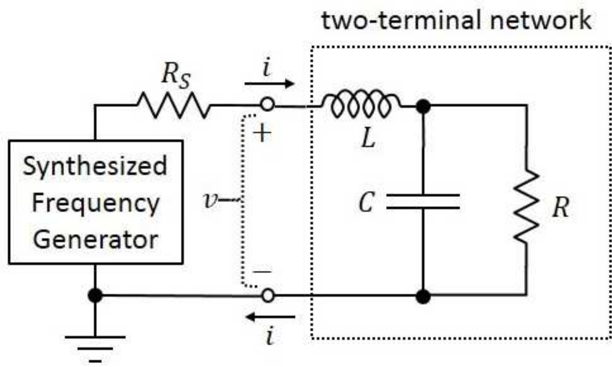

Figure 4: Testing the two-terminal network

Place the two-terminal network of Figure 3 with parameters of Table 3 into the larger circuit shown in Figure 4. For the resistor R_S use 2 kΩ. Set the synthesized frequency generator to produce a 2-kHz sinewave with an amplitude of 4 V.

Connect channel 1 of the oscilloscope to observe the output of the synthesized frequency generator. Connect channel 2 to observe the voltage v (across the terminals of the two-terminal network). Use AC coupling for both channels. Set both channels 1 and 2 to the same vertical scale. From the oscilloscope display, estimate the phase of v relative to the source voltage. Make sure you assign the correct sign to this phase. Use an AC voltmeter to measure the rms value of v. Convert this to an amplitude. You now have an experimental determination of the phasor that represents v.

We now want to make an experimental determination of the phasor that represents i. We could do this by reordering the components in the circuit, but instead we will do this with a "math trace" on the oscilloscope display. You can introduce a math trace by using the math menu. On this menu, request the display of the channel-1 voltage minus the channel-2 voltage. Make sure that both channels 1 and 2 are using AC coupling. It is essential that the vertical scales for channels 1 and 2 be the same. Notice that this difference display represents the voltage across the resistor R_S, which equals $i \cdot R_S$.

Set the oscilloscope to display only the source voltage (channel 1) and the (math) difference voltage. In other words, remove the channel-2 display from the oscilloscope screen. (The channel-2 connection to the oscilloscope should remain in place, because the math mode needs this signal to calculate the difference voltage. But we only want to see the source voltage and the difference voltage on the screen.) From the oscilloscope display, estimate the phase of $i \cdot R_S$ relative to the source voltage. You should recognize that the phase of i is the same as the phase of $i \cdot R_S$, since R_S is a real, positive constant. Make sure you assign the correct sign to this phase.

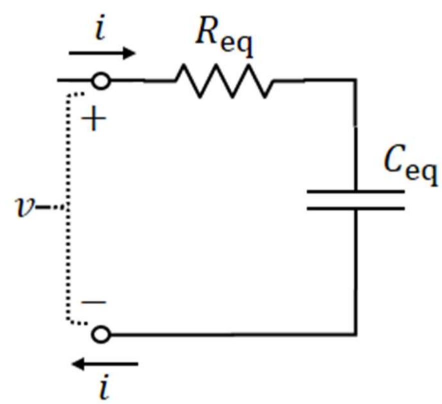

Figure 5: Equivalent network (for 2 kHz)

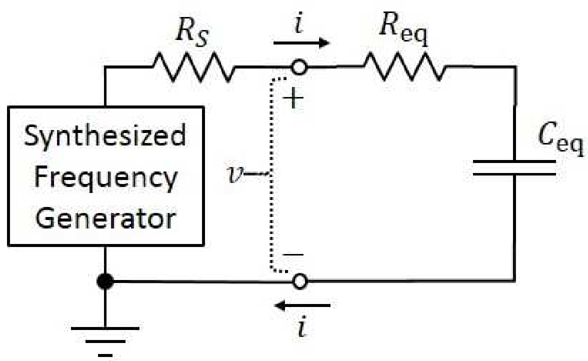

Figure 6: Testing the equivalent, frequency-specific network

Use an AC ammeter to measure the rms value of i. Convert this to an amplitude. You now have an experimental determination of the phasor that represents i.

You will now replace the original two-terminal network with its equivalent network, valid for a frequency of 2 kHz. This equivalent network is shown in Figure 5. The placement of this equivalent network into the larger circuit is shown in Figure 6. As before, set R_S to 2 kΩ and the synthesized frequency generator for a 2-kHz sinewave having amplitude 4 V.

Using the procedure explained above, make an experimental determination of the phasors representing v and i for the circuit of Figure 6.

Lab Report

For the *RC* voltage divider, do your measured results agree, at least approximately, with predictions?

For the *RL* voltage divider, do your measured results agree, at least approximately, with predictions?

When the equivalent network of Figure 5 replaces the two-terminal *RLC* network of Figure 3 in a larger circuit that is excited by a *2-kHz sinewave*, do v and i remain unchanged?

Finally, here are two hypothetical questions. (You did not actually test these circumstances.)

If the equivalent network of Figure 5 replaces the two-terminal *RLC* network of Figure 3 in a larger circuit that is excited by a *1-kHz* sinewave, will v and i remain unchanged?

If the equivalent network of Figure 5 replaces the two-terminal *RLC* network of Figure 3 in a larger circuit that is excited by a 2-kHz *square-wave*, will v and i remain unchanged?